Dr. Ahmed Soil Mechanics Notes Chapter Five (Permeability and Seepage through Soil)

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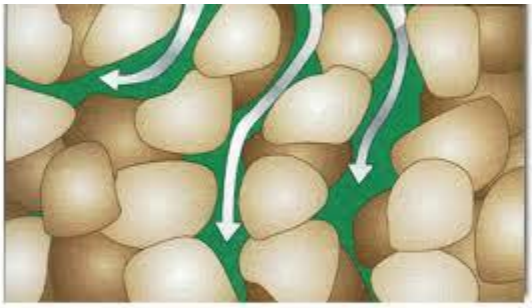

CHAPTER FIVE

PERMEABILITY AND SEEPAGE THROUGH SOIL

Lecture Notes

Soil Mechanics

3rd Class

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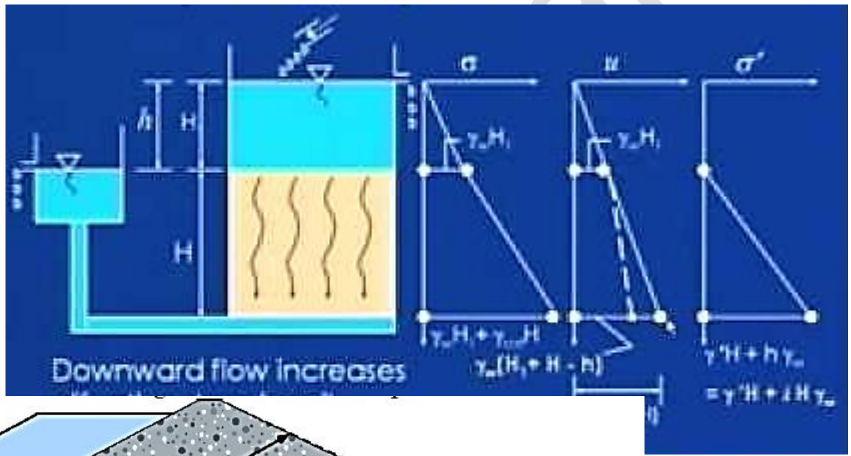

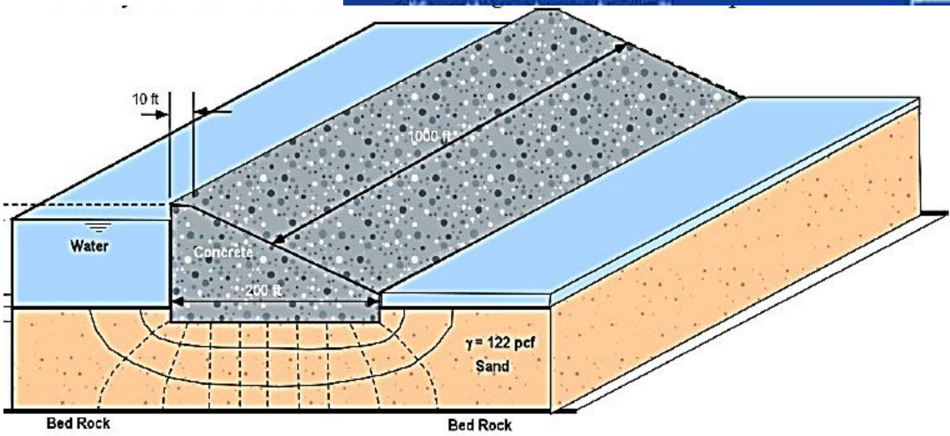

Assistant Prof.

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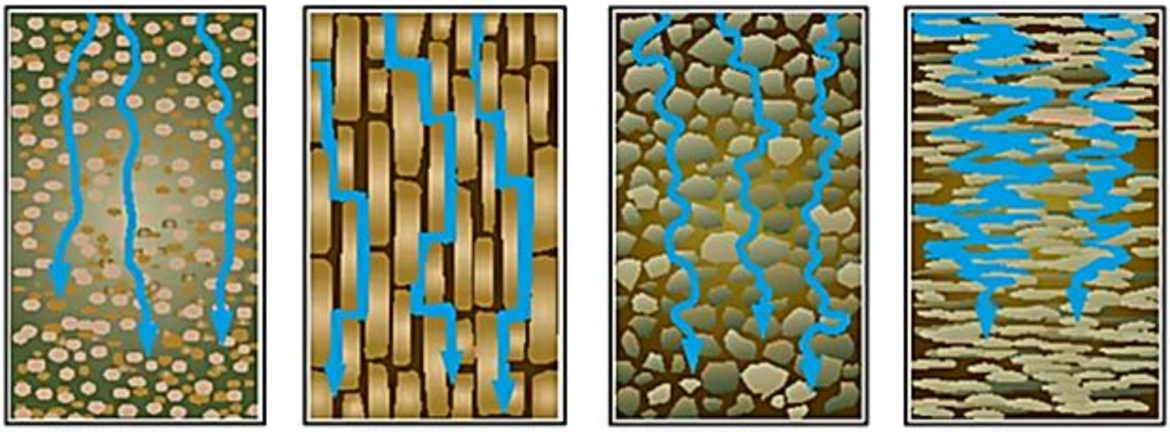
CHAPTER FIVE

PERMEABILITY AND SEEPAGE THROUGH SOIL

5.1 Introduction

Soils are permeable due to the existence voids through which water can flow from points of high energy to points of low energy. The study of the flow of water through permeable soil media is important in soil mechanics.



- To evaluate the rate of flow under dams and hydraulic structures.
- To determine the shear strength of soil when there is a flow of water.
- To determine the settlement.
- Investigating problems involving the pumping of water for underground construction, and for making stability analyses of earth dams and earth-retaining structures that are subject to seepage forces.

One of the major physical parameters of a soil that controls the rate of seepage through it is ***coefficient of permeability (hydraulic conductivity)***

Permeability: The property of soils that allows water to pass through them at some rate.

5.2 Hydraulic Gradient

When water flows through a saturated soil mass, there is a certain resistance to the flow because of the presence of solid matter. However, the laws of fluid mechanics which are applicable for the flow of fluids through pipes are also applicable to flow of water through soils. As per Bernoulli's equation, the total head at any point in water under steady flow condition may be expressed as:

Total head = pressure head + velocity head + elevation head

$$h_t = \frac{u}{\gamma_w} + \frac{v^2}{2g} + Z$$

↑ Pressure head ↑ Velocity head ↑ Elevation head

Where h_t = total head,

u = pore water pressure, and γ_W = unit weight of water, the term $\frac{u}{\gamma_w} = h_p$, (m)

υ = velocity, g = acceleration due to gravity, this value is very small = 0

Z = elevation head, (m) = h_e

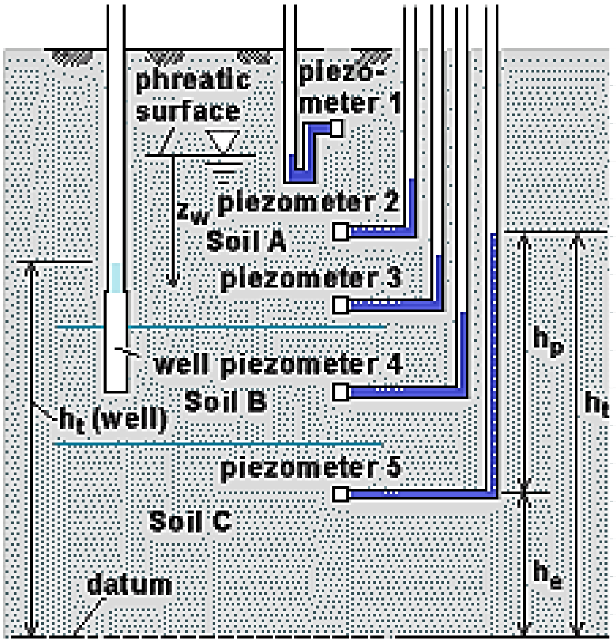
$h_t = h_p + h_e$

Elevation head (h_e): is the vertical distance from a datum line to the point considered (above or below a datum plane).

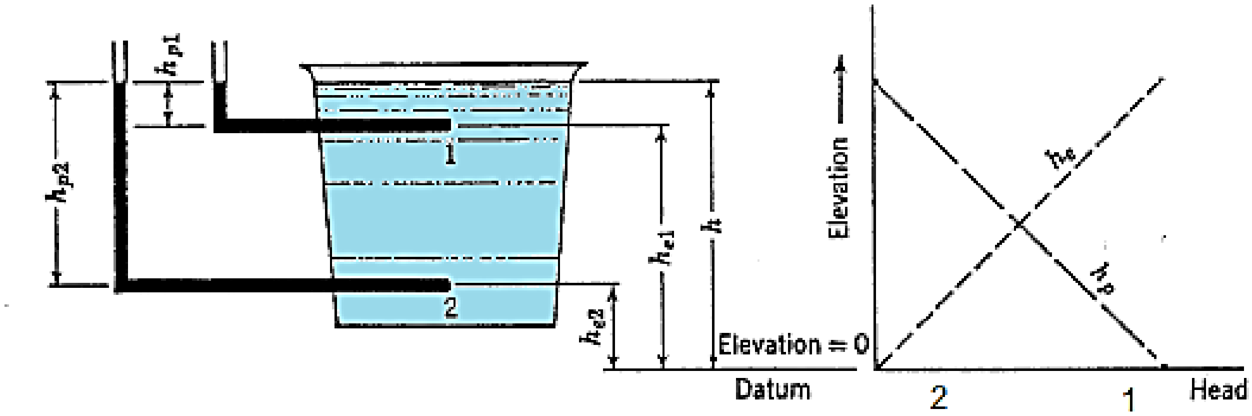
Pressure head (h_p): is the height of the vertical column of water in the piezometer installed at that point.

Piezometer: is a device used to measure liquid pressure in a system by measuring the height to which a column of the liquid rises against gravity, or a device which measures the pressure of groundwater at a specific point.

Total head (h_t) = elevation head + pressure head

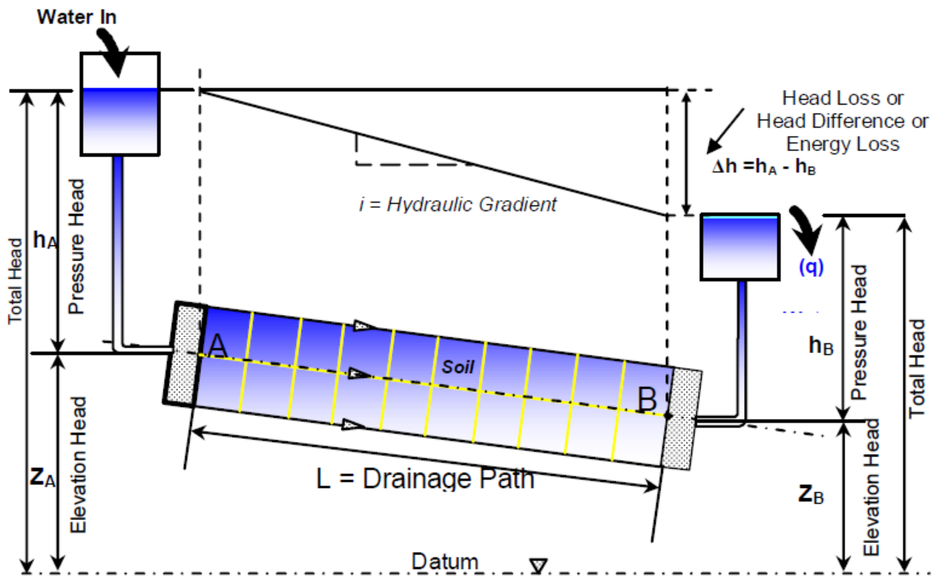


In the example below, there is a difference in elevation heads and pressure head, and no difference in total head. So that there is no flow.



Point	Elevation head	Pressure head	Total head
1	h_{e1}	h_{p1}	$h_{e1} + h_{p1} = h_t$
2	h_{e2}	h_{p2}	$h_{e2} + h_{p2} = h_t$

In the example below, there is a difference in elevation heads and pressure head, and also difference in total head. So that there is flow.



So that:

<u>The flow between any two points depends only on the total difference in total head.</u>

<u>Any elevation head can be selected from datum as the base of elevation head.</u>

Hydraulic gradient

$$i = \frac{\Delta h}{L}$$

Where

i = hydraulic gradient

L = distance between A and B (the length of flow over which loss of head occurred)

5.3 Darcy's Law

In 1856, Darcy published a simple equation for discharge velocity of water through saturated soils, which may express as

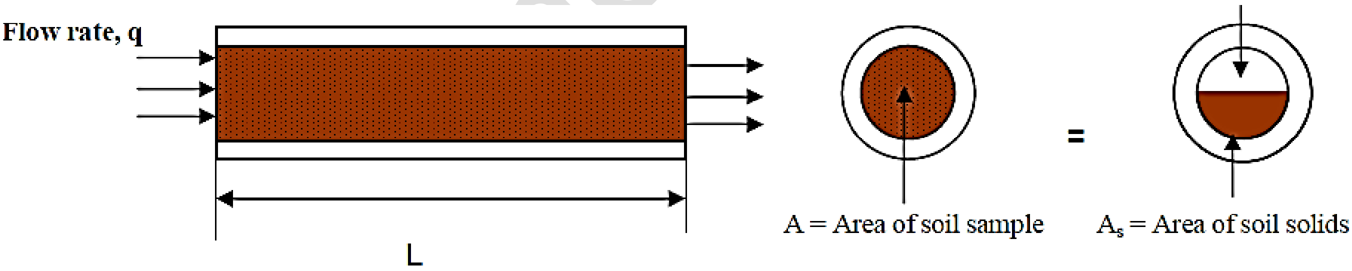
v* = *ki

Where

v = discharge velocity = quantity of water flowing in unit time through a unit gross – sectional area of soil at right angles to the direction of flow. (m/sec)

k = coefficient of permeability

v = is based on the gross – sectional area of the soil, however the actual velocity of water (seepage velocity, v_s) through the void spaces, is higher than v, this can be derived as follows:



If the flow rate q, then

$$q = vA = A_v . v_s \qquad A = A_v + A_s \qquad \therefore q = v(A_v + A_s) = A_v . v_s$$

so

$$v_s = \frac{v(A_v + A_s)}{A_v} = \frac{v(A_v + A_s)L}{A_v L} = \left(\frac{v(V_v + V_s)}{V_v}\right) \div Vs$$

$$v_s = v\left[\frac{1 + \frac{V_v}{V_s}}{\frac{V_v}{V_s}}\right] = v\left(\frac{1+e}{e}\right) = \frac{v}{n}$$

5.4 Coefficient of Permeability (Hydraulic Conductivity)

It is defined as the rate of flow per unit area of soil under unit hydraulic gradient, it has the dimensions of velocity (L/T) such (cm/sec or ft/sec). It depends on several factors as follows:

1. Shape and size of the soil particles.
2. Distribution of soil particles and pore spaces.
3. Void ratio, permeability increases with increase of void ratio
4. The degree of saturation, permeability increases with the increase of the degree of saturation.
5. The composition of soil particles.
6. Soil structure
7. Fluid properties

(k) Varies widely for different soils, as shown in the table

Typical values of permeability coefficient (k)	
Soil type	k (mm/sec)
Coarse gravel	10 to 10^{3}
Fine gravel, coarse and medium sand	10^{-2} to 10
Fine sand, loose silt	10^{-4} to 10^{-2}
Dense silt, clayey silt	10^{-5} to 10^{-4}
Silty clay, clay	10^{-8} to 10^{-5}

5.5 Tests to Find the Hydraulic Conductivity

5.5.1 Laboratory Tests

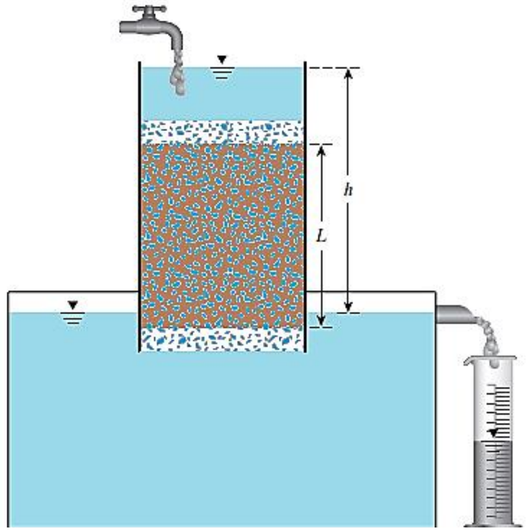
The four most common laboratory methods for determining the permeability coefficient of soils are the following:

1. Constant – head test.
2. Falling – head test.
3. Indirect determination from consolidation test
4. Indirect determination by the horizontal capillary test.

Constant – head test

In this test a direct measure of permeability using Darcy's Law

It's suitable for cohesionless soils with permeability > 10^{-4} cm/sec



A typical arrangement of the constant-head permeability test is shown in Figure. In this type of laboratory setup, the water supply at the inlet is adjusted in such a way that the difference of head between the inlet and the outlet remains constant during the test period.

After a constant flow rate is established, water is collected in a graduated flask for a known duration.

The total volume of water collected may be expressed as:

$$Q = Avt = A(ki)t$$

Where:

Q = volume of water collected

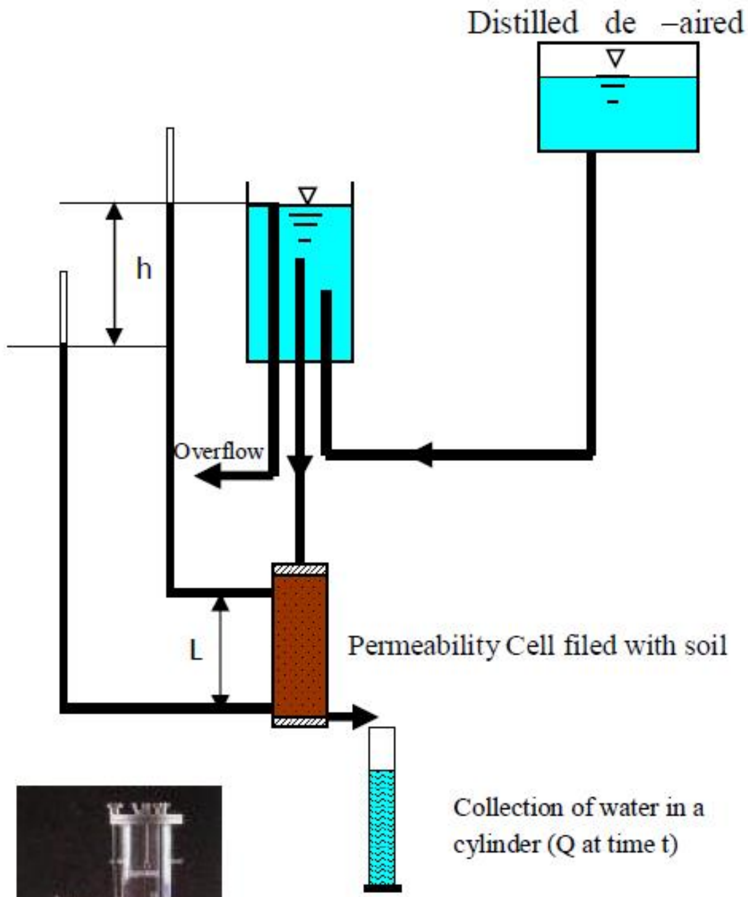
A = area of cross-section of the soil specimen

t = duration of water collection

Since $i = \dfrac{h}{L}$

Where L = length of the specimen, so that

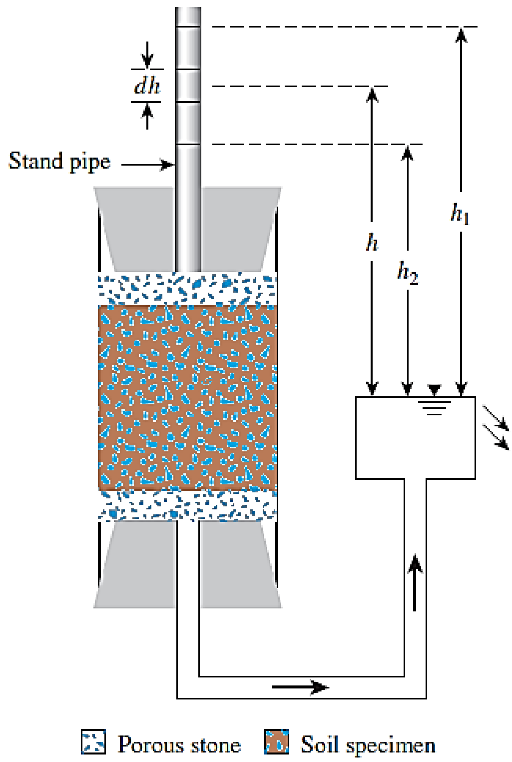
or $k = \dfrac{QL}{Aht}$



Permeability Cell with

Falling-Head Test

A typical arrangement of the falling-head permeability test is shown in Figure. Water from a standpipe flows through the soil. The initial head difference h_1 at time t = 0 is recorded, and water is allowed to flow through the soil specimen such that the final head difference at time $t = t_2$ is h_2.



The rate of flow of the water through the specimen at any time t can be given by

$$q = k\frac{h}{L}A = -a\frac{dh}{dt}$$

Where:

q = flow rate

a = cross-sectional area of the standpipe

A = cross-sectional area of the soil specimen

Rearrangement of Eq. gives

$$dt = \frac{aL}{Ak}\left(-\frac{dh}{h}\right)$$

Integration of the left side with limits of time from 0 to t and the right side with limits of head difference from h_1 to h_2 gives:

$$t = \frac{aL}{Ak}\log_e\frac{h_1}{h_2} \quad \text{or} \quad k = 2.303\frac{aL}{At}\log_{10}\frac{h_1}{h_2}$$

Example 5.1

Refer to the constant-head permeability test arrangement. A test gives these values:

L = 30 cm, A = area of the specimen = 177 cm^2, Constant-head difference, h = 50 cm, Water collected in a period of 5 min = 350 cm^3

Calculate the hydraulic conductivity in cm/sec.

Solution

$$k = \frac{QL}{Aht}$$

Given Q = 350 cm^3, L = 30 cm, A = 177 cm^2, h = 50 cm, and t = 1.0 min, so

$$k = \frac{350 * 30}{177 * 50 * 1 * 60} = 1.98 * 10^{-2} cm/sec$$

Example 5.2

For a falling-head permeability test, the following values are given:

Length of specimen = 200 mm, Area of soil specimen = 1000 mm^2, Area of standpipe = 40 mm^2, Head difference at time t = 0 = 500 mm, Head difference at time t = 280 sec = 300 mm.

Determine the hydraulic conductivity of the soil in cm/sec.

Solution

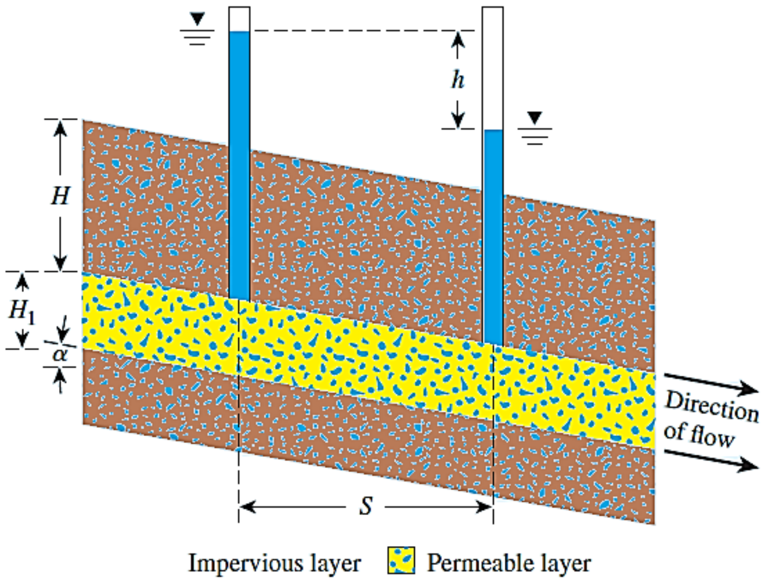
$$k = 2.303 \frac{aL}{At} \log_{10} \frac{h_1}{h_2}$$

For a given a = 40 mm^2, L = 200 mm, A = 1000 mm^2, t = 180 sec, h_1 = 500 mm, and h_2 = 300 mm.

$$k = 2.303 \frac{40 * 200}{1000 * 280} log_{10} \frac{500}{300} = 1.46 * 10^{-2}\ cm/sec$$

Example 5.3

Find the flow rate in m^3/sec/m length (at right angles to the cross-section shown) through the permeable soil layer shown in Figure, given H = 8 m, H1 =3 m, h = 4 m, S = 50 m, a = 8°, and k = 0.08 cm/sec.



Solution

$$\text{Hydraulic gradient } (i) = \frac{h}{\frac{S}{\cos\alpha}}$$

$$q = kiA = k\left(\frac{h\cos\alpha}{S}\right)(H_1 \cos\alpha \times 1)$$

$$= (0.08 \times 10^{-2}\ \text{m/sec})\left(\frac{4\cos 8°}{50}\right)(3\cos 8° \times 1)$$

$$= \mathbf{0.19 \times 10^{-3}\ m^3/sec/m}$$

5.5.2 Empirical Equations

For granular soil

Hazen (1930)

$$k\ (\text{cm/sec}) = cD_{10}^2$$

where c = a constant that varies from 1.0 to 1.5

D_{10} = the effective size, in mm

Carrier (2003)

$$k = 1.99 \times 10^4 \left[\frac{100\%}{\sum \frac{f_i}{D_{li}^{0.404} \times D_{si}^{0.595}}}\right]^2 \left(\frac{1}{SF}\right)^2 \left(\frac{e^3}{1+e}\right)$$

where f_i = fraction of particles between two sieve sizes, in percent (Note: larger sieve, *l*; smaller sieve, *s*)

$$D_{(\text{av})i}(\text{cm}) = [D_{li}(\text{cm})]^{0.5} \times [D_{si}(\text{cm})]^{0.5}$$, SF = shape factor

Amer and Awad (1974)

$$k(\text{cm/sec}) = 35\left(\frac{e^3}{1+e}\right)C_u^{0.6}(D_{10})^{2.32}$$

where k is in cm/sec

C_u = uniformity coefficient

D_{10} = effective size (mm)

For cohesive soil

$$\log k = \log k_o - \frac{e_o - e}{C_k}$$

where k_o = in situ hydraulic conductivity at a void ratio e_o

k = hydraulic conductivity at a void ratio e

C_k = hydraulic conductivity change index

5.5.3 Field tests

There are many useful methods to determine the (k) in the field such as:

1. Pumping from wells
2. Borehole test
3. Open – end test
4. Packer test
5. Variable – head tests by means of piezometer observation well

Pumping from Wells

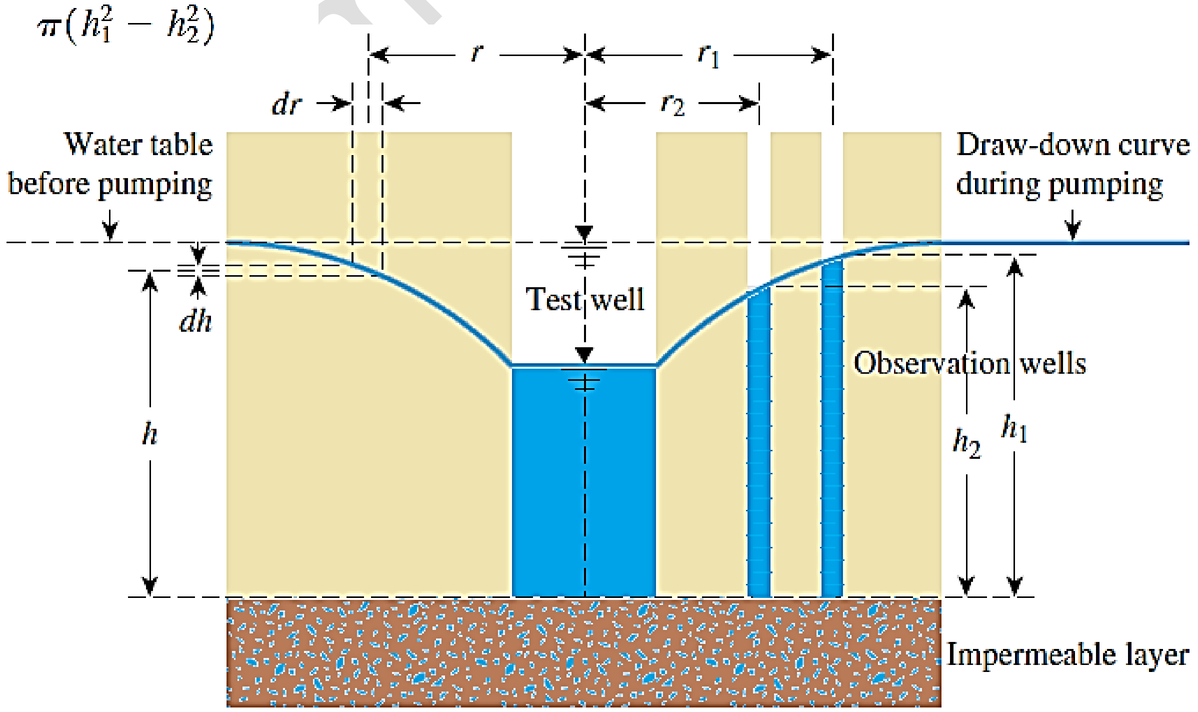
In the field, the average hydraulic conductivity of a soil deposit in the direction of flow can be determined by performing pumping tests from wells. The figure shows a case where the top permeable layer, whose hydraulic conductivity has to be determined is underlain by an impermeable layer.

During the test, water is pumped out at a constant rate from a test well that has a perforated casing. Several observation wells at various radial distances are made around the test well. Continuous observations of the water level in the test well and in the observation wells are made after the start of pumping until a steady state is reached. The steady state is established when the water level in the test and observation wells becomes constant. The expression for the rate of flow of groundwater into the well, which is equal to the rate of discharge from pumping, can be written as:

$$q = k\left(\frac{dh}{dr}\right)2\pi r h$$

$$\int_{r_2}^{r_1} \frac{dr}{r} = \left(\frac{2\pi k}{q}\right) \int_{h_2}^{h_1} h\, dh$$

$$k = \frac{2.303 q \log_{10}\left(\frac{r_1}{r_2}\right)}{\pi(h_1^2 - h_2^2)}$$



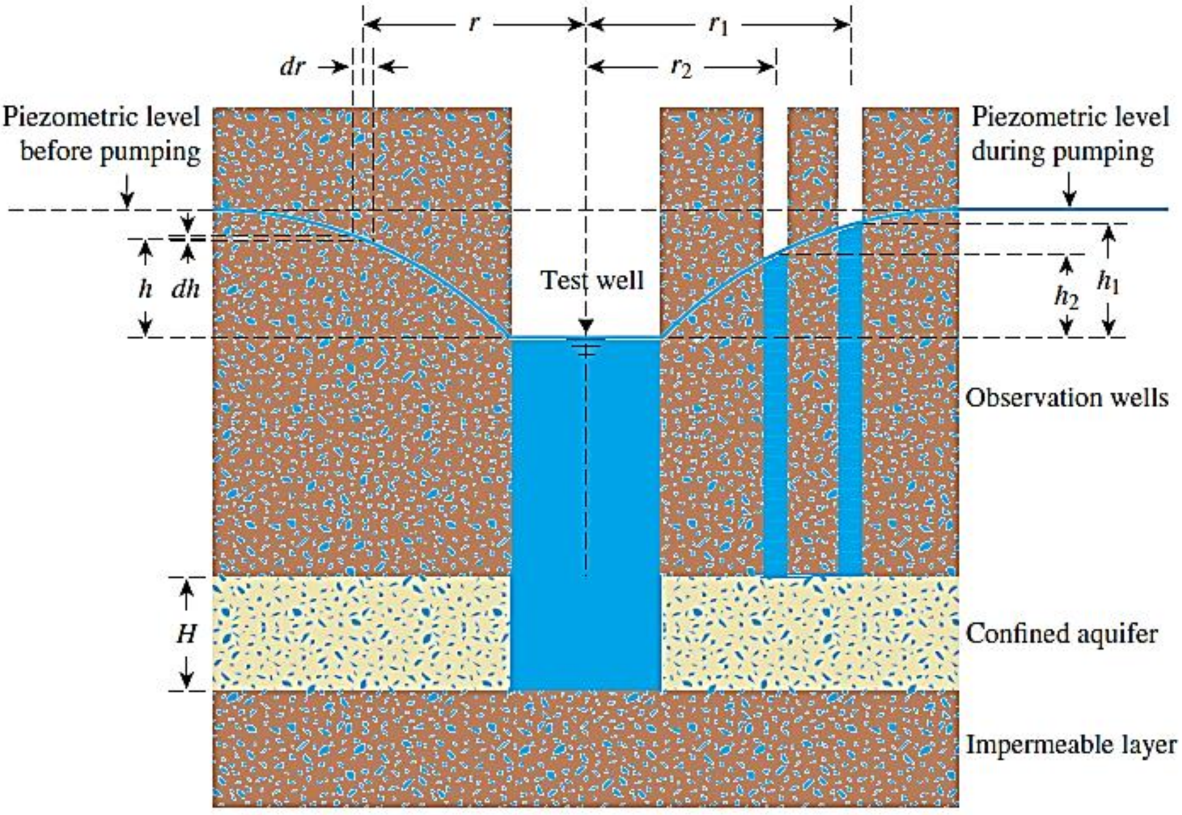
From field measurements, if q, r_1, r_2, h_1, and h_2 are known, the hydraulic conductivity can be calculated from the simple relationship.

The average hydraulic conductivity for a confined aquifer can also be determined by conducting a pumping test from a well with a perforated casing that penetrates the full depth of the aquifer and by observing the piezometric level in a number of observation wells at various radial distances (see figure below). Pumping is continued at a uniform rate q until a steady state is reached. Because water can enter the test well only from the aquifer of thickness H, the steady state of discharge is

$$q = k\left(\frac{dh}{dr}\right)2\pi r H$$

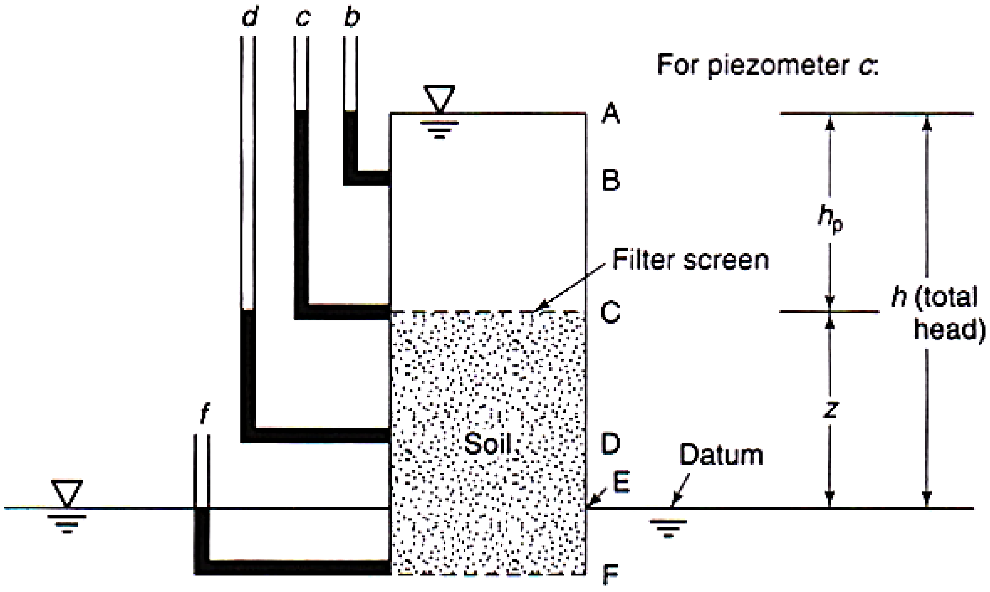
$$\int_{r_2}^{r_1}\frac{dr}{r} = \int_{h_2}^{h_1}\frac{2\pi k H}{q}\,dh$$

$$k = \frac{q\ \log_{10}\left(\frac{r_1}{r_2}\right)}{2.727H(h_1 - h_2)}$$



Aquifer: An underground geological formation able to store and yield water

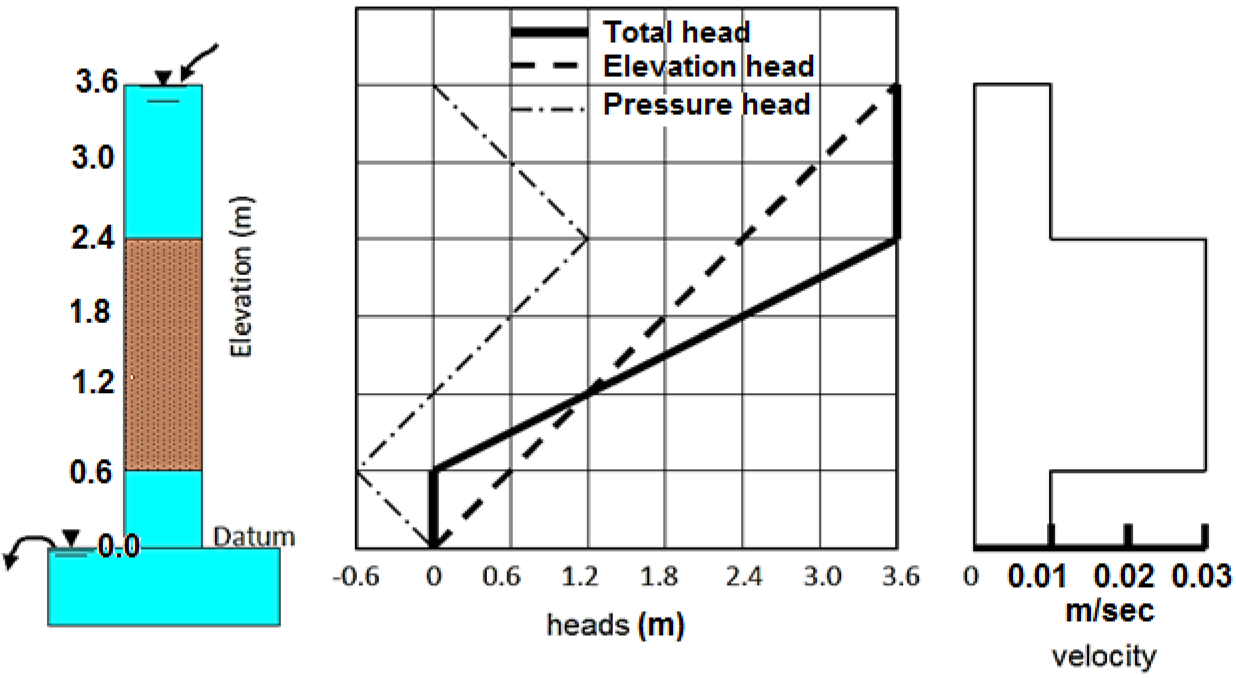
5.6 Heads and One Dimensional Flow



Point	Pressure Head	Elevation Head	Total Head	Head Loss through Soil
B	AB	BE	AE	0
C	AC	CE	AE	0
D	CD	DE	CE	½AE
F	EF	−EF	0	AE

Example 5.4

Draw elevation head, pressure head, and total head for the system shown and then find the velocity of the seepage in soil, if k = 0.005 m/sec and n = 0.33



$v = k.i = 0.005 \times \frac{3.6}{1.8} = 0.01$ m/sec at the entrance and the exit parts of the tube.

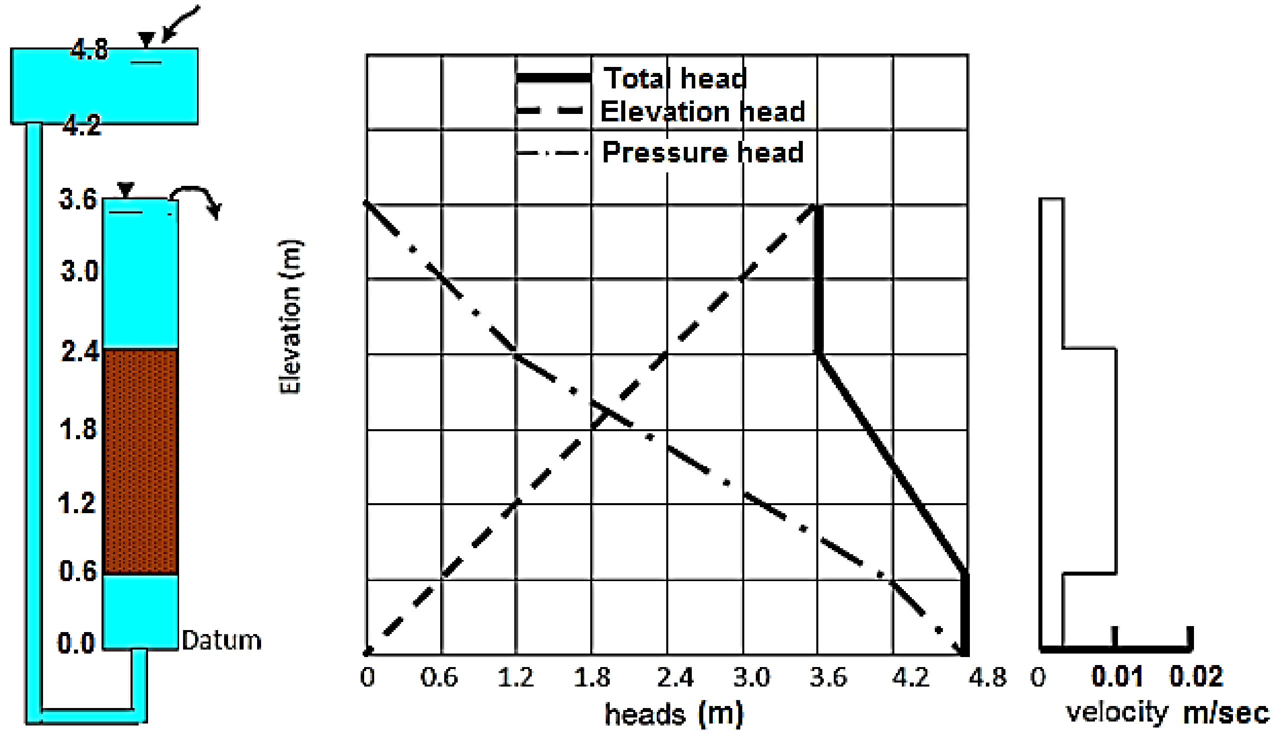
$\therefore v_s = \frac{v}{n} = \frac{0.01}{0.33} = 0.03$ m/sec through the soil sample

<u>The flow is downward because the total head of the upper point is larger than lower point in the sample</u>

<u>Example 5.5</u>

Draw elevation head, pressure head, and total head for the system shown and then find the velocity of the seepage in soil, if k = 0.005 m/sec and n = 0.33

<u>Solution</u>

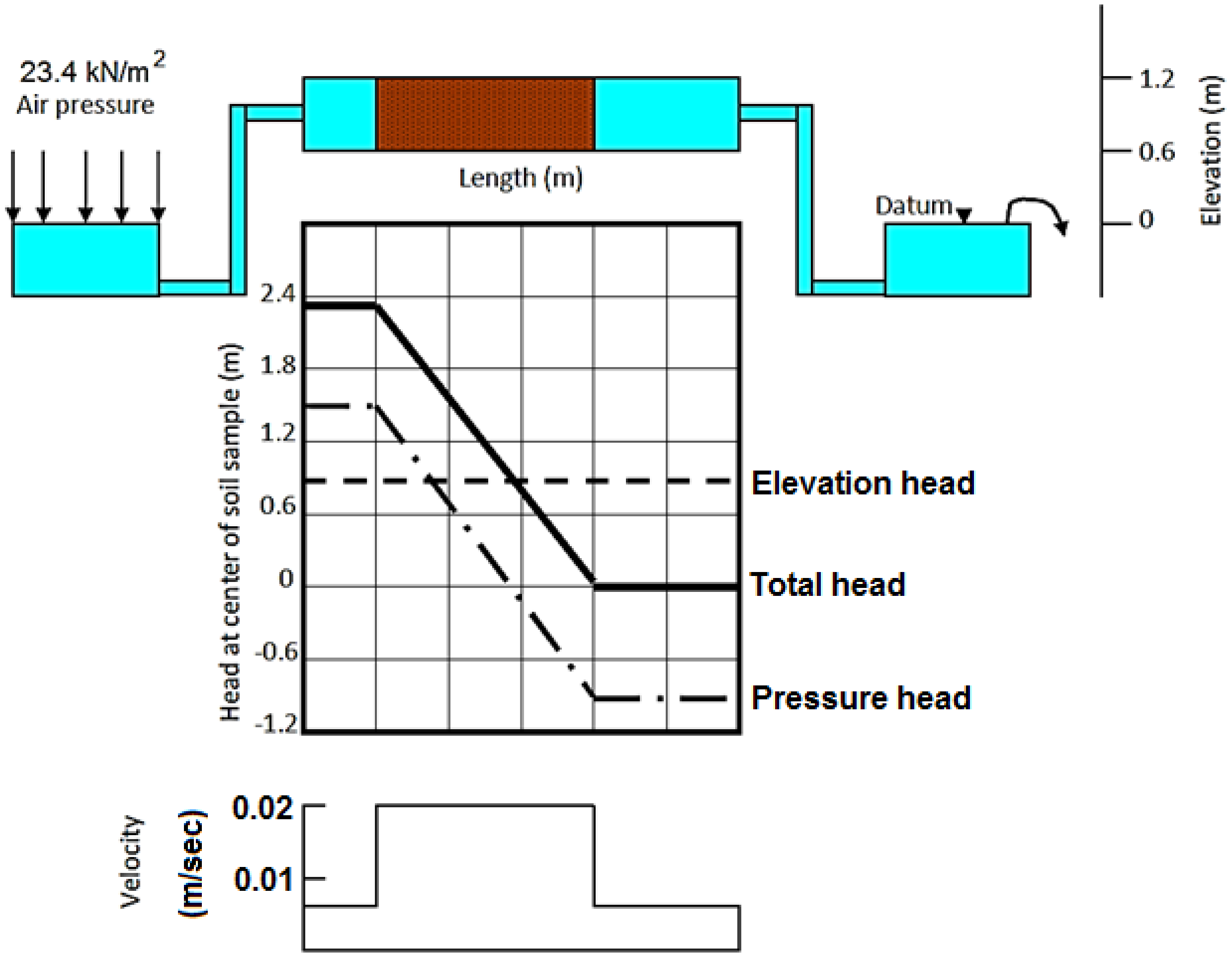


$v = k.i = 0.005 \times \frac{4.8 - 3.6}{1.8} = 0.0033 \Rightarrow v_s = \frac{v}{n} = \frac{0.0033}{0.33} = 0.01$ m/sec

<u>The flow is upward because the total head of the lower point is larger than upper point in the sample</u>

Example 5.6

Draw elevation head, pressure head, and total head for the system shown and then find the velocity of the seepage in soil, if k = 0.05 m/sec and n = 0.33



In this case, the air pressure will produce the required head for horizontal flow. Thus

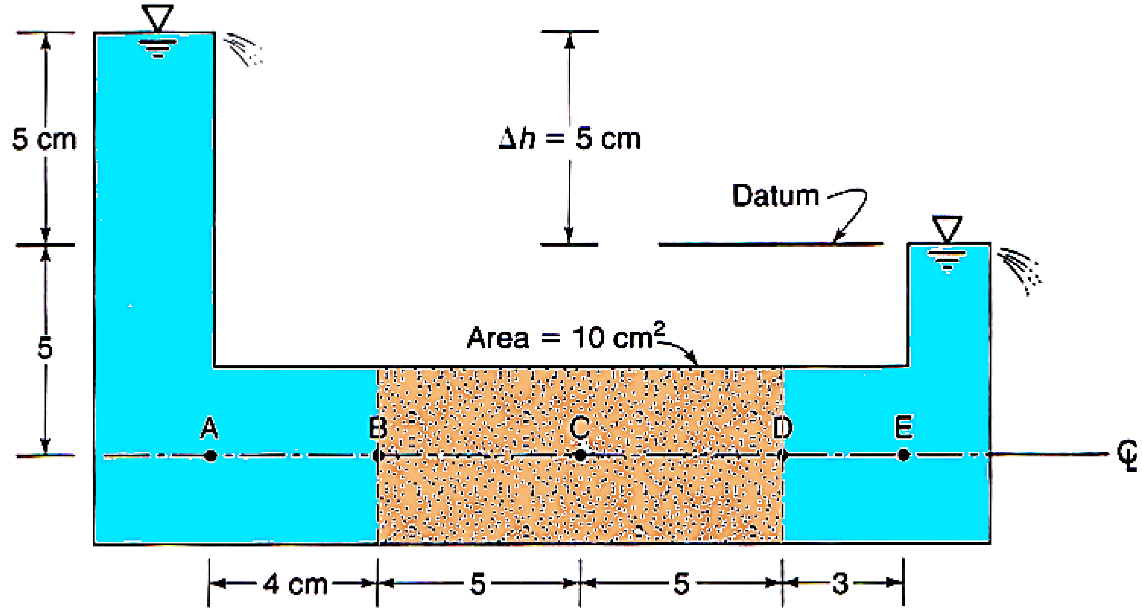
$$Total\ head\ loss = \frac{23.4}{9.81} = 2.385\ m.$$

$$v = k.i = 0.005\frac{2.385}{1.8} = 0.00663 \Rightarrow v_s = \frac{v}{n} = \frac{0.00663}{0.33} = 0.02\ m/\sec$$

<u>The flow is horizontal to left because the total head in the right point is higher than the left point</u>

Example 5.7

Draw elevation head, pressure head, and total head for the system shown and then find the heads for point c.

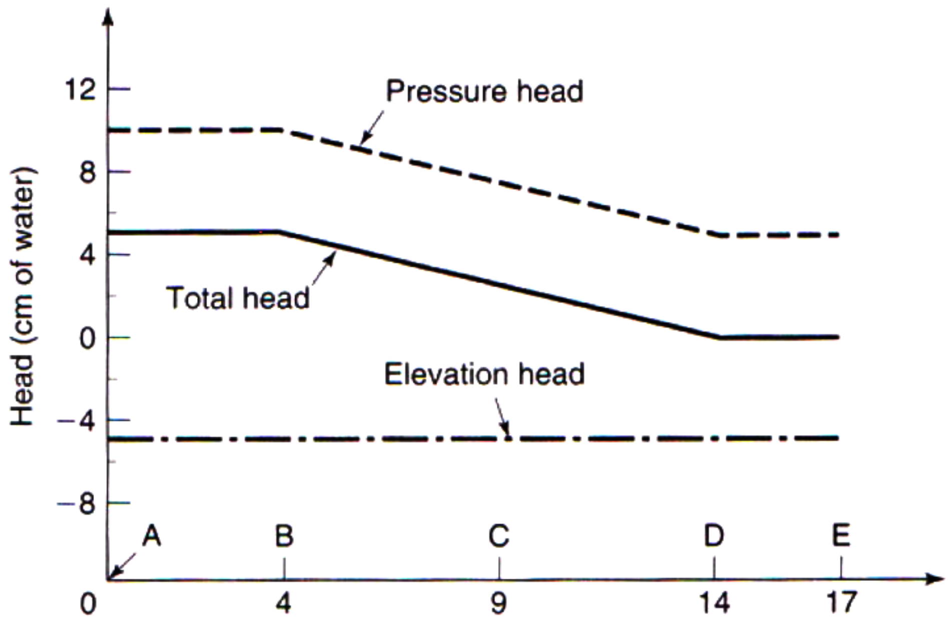


(a)

Solution

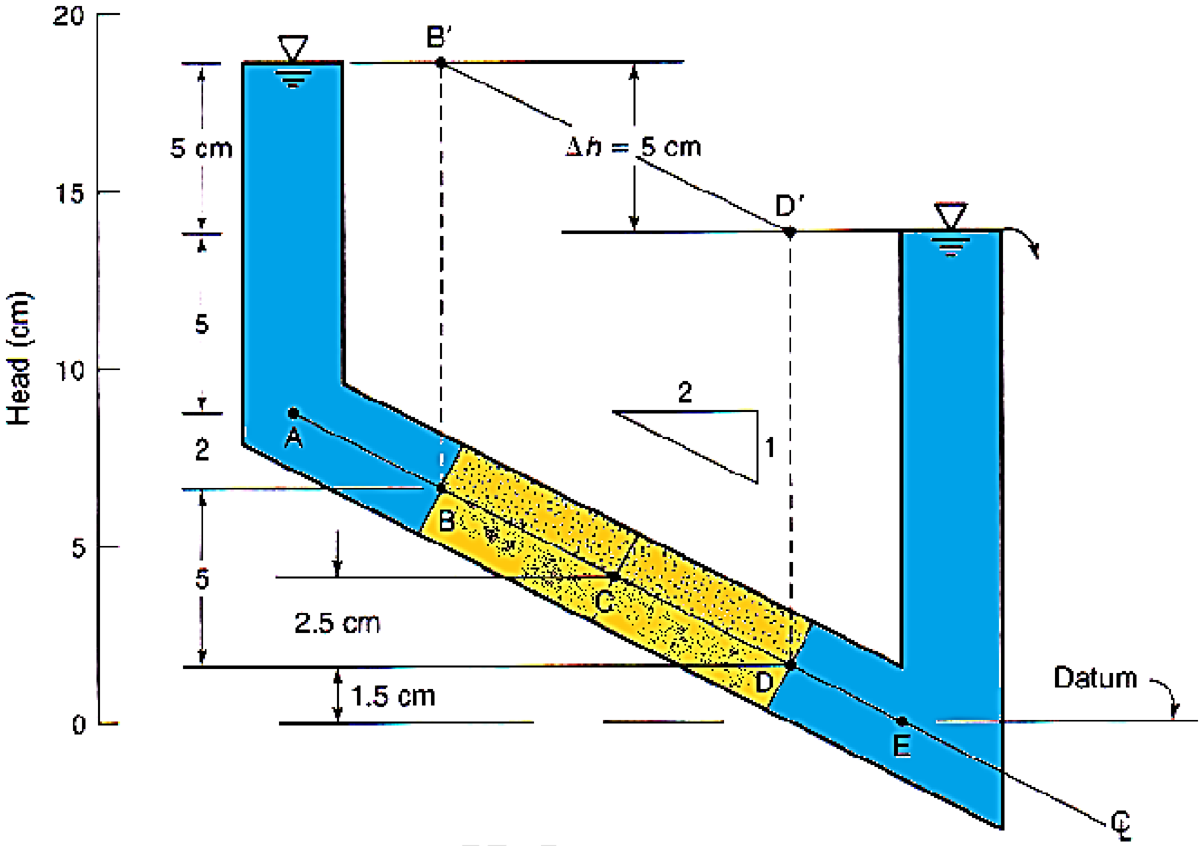
(1) Find the h_e for all points, (2) find hp for points A, B, D, E (3) find h_t for points A, B, D, E. (4) from figure find hp and h_t for point c.

Point	h_e (cm)	h_p (cm)	h_t (cm)
A	-5	10	5
B	-5	10	5
C	-5	(7.5)	(2.5)
D	-5	5	0
E	-5	5	0



Example 5.8

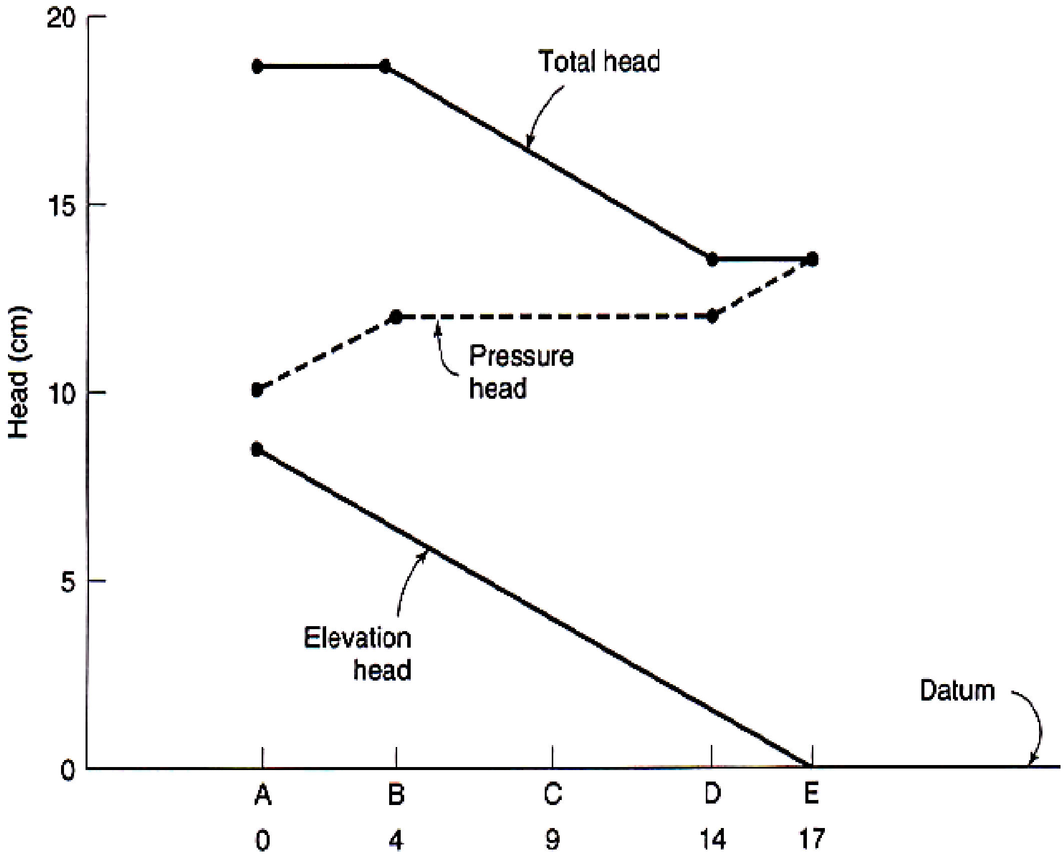
Draw elevation head, pressure head, and total head for the system shown and then find the heads for point c



Solution

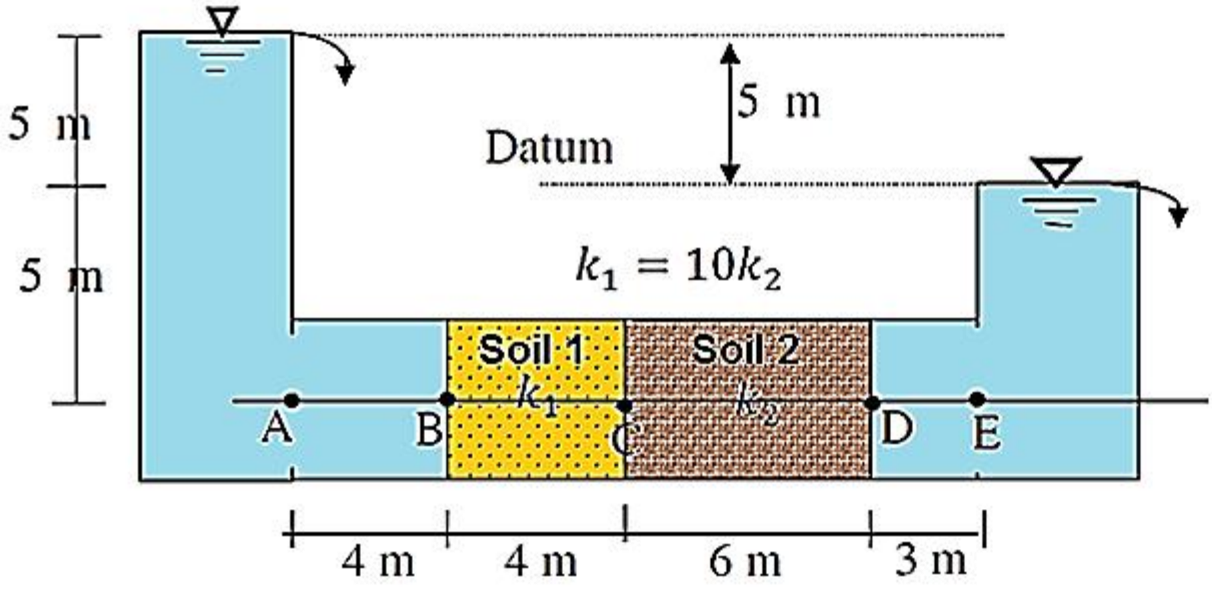
(1) Find the h_e for all points, (2) find hp for points A, B, D, E (3) find h_t for points A, B, D, E. (4) from figure find hp and h_t for point c.

Point	h_e (cm)	h_p (cm)	h_t (cm)
A	8.5	10	18.5
B	6.5	12	18.5
C	4.0	12	16
D	1.5	12	13.5
E	0	13.5	13.5



Example 5.9

For the setup shown, Find total head (h_t), Elevation head (h_e) and Pressure head (h_p) for the soil the setup shown.



Solution

The quantity of flow in soil (1) has to be the same as in soil (2)

$q_1 = k_1 i_1 A_1 = q_2 = k_2 i_2 A_2$

Since the area are the same so that

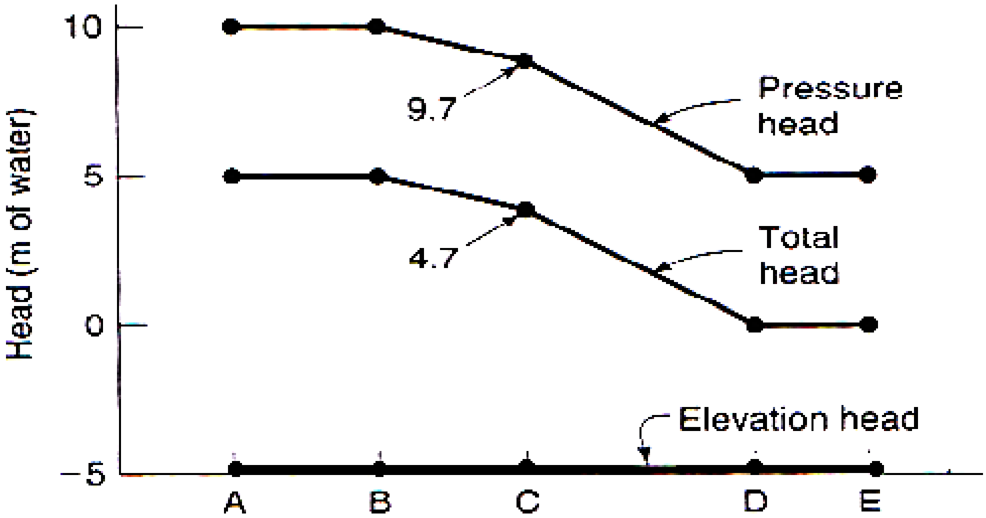
$k_1 i_1 = k_2 i_2$, $k_1 = 10k_2$

$$10\,k_2\frac{\Delta h_{t1}}{l_1} = k_2\frac{\Delta h_{t2}}{l_2},\quad 2.5\Delta h_{t1} = 0.167\Delta h_{t2},\quad \Delta h_{t1} = 0.0668\Delta h_{t2}$$

The total head lost (Δh_t) = $\Delta h_{t1} + \Delta h_{t2}$, from figure = 5

$5 = 0.0668\Delta h_2 + \Delta h_2 \rightarrow \Delta h_2 = 4.67m$, $\Delta h_1 = 0.33$

Thus h_t at point c = 4.67m and $h_p = 4.67 - (-5) = 9.67m$



Point	h_e (cm)	h_p (cm)	h_t (cm)
A	-5	10	5
B	-5	10	5
C	-5	9.67	4.67
D	-5	5	0
E	-5	5	0

Because the permeability of soil (2) is much less than soil (1) thus the most head lost in soil (2)

Example 5.10

For Setup shown: Soil I, A= 0.37 m^2, n=0.5 and k= 1 m/sec, Soil II, A= 0.186 m^2, n=0.5 and k=0.5 m/sec. Find total head (h_t), Elevation head (h_e) and Pressure head (h_p).

Solution

1- Chose points at elevation 1, 2, 3, 4, 5
2- Draw the elevation head for the points above
3- Draw the pressure head 1, 2, 4, and 5, the pressure head for point 3 cannot be drawn.

4- Draw total head for points 1, 2, 4, and 5 the total head for point 4 cannot be drawn.

5- To find the total head at point 3:

$q_I = k_I i_1 A_I = q_{II} = k_{II} i_{II} A_{II}$

$$1.0 * \frac{\Delta h_{tI}}{1.2} * 0.37 = 0.5 * \frac{\Delta h_{tII}}{0.6} * 0.186$$

$0.31\Delta h_{tI} = 0.155\ \Delta h_{tII} \rightarrow \Delta h_{tI} = 0.5\ \Delta h_{tII}$

$\Delta h_{tI} + \Delta h_{tII} = 3.6 - 0 = 3.6m$

$0.5\ \Delta h_{tII} + \Delta htII = 3.6 \rightarrow \Delta h_{tII} = 2.4 \rightarrow \Delta h_{tI} = 3.6 - 2.4 = 1.2m$

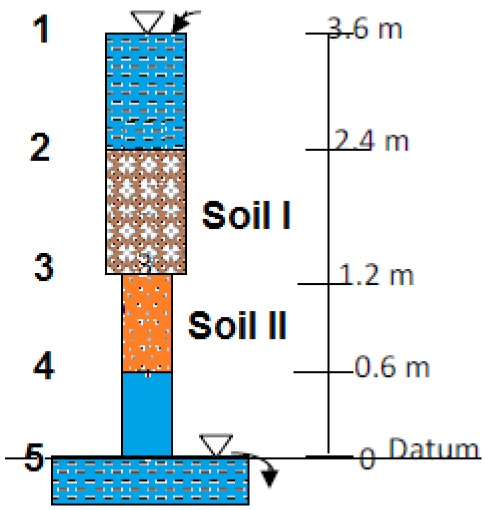
Thus h_{tI} at point 3 is 2.4 and h_{pI} = 2.4 -1.2 = 1.2m

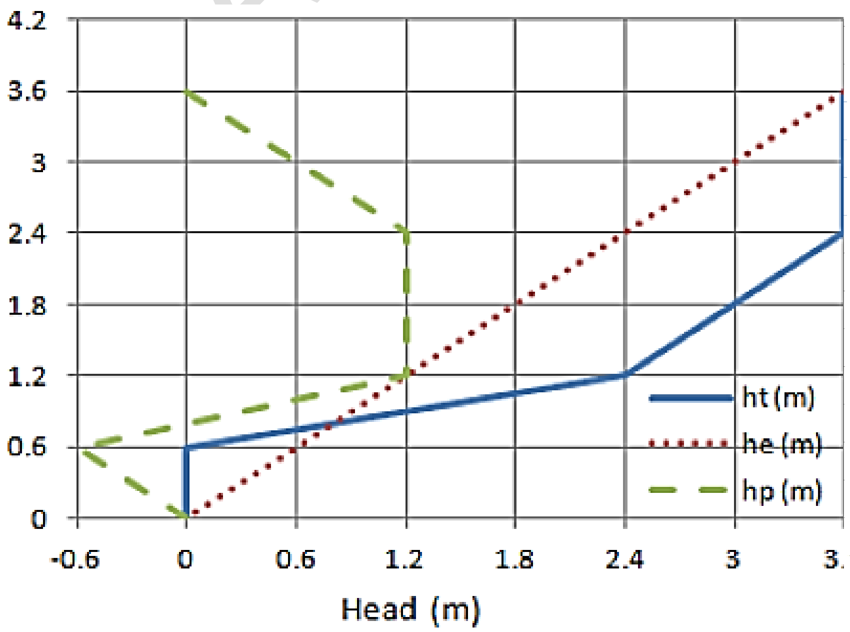
The velocity above soil I = $ki = 1 * \frac{1.2}{2.4-1.2} = 1\ m/sec$

The velocity through soil I = v/n = 1/0.5 = 2 m/sec

The velocity below soil II = $ki = 1 * \frac{2.4}{1.2-0.6} = 2\ m/sec$

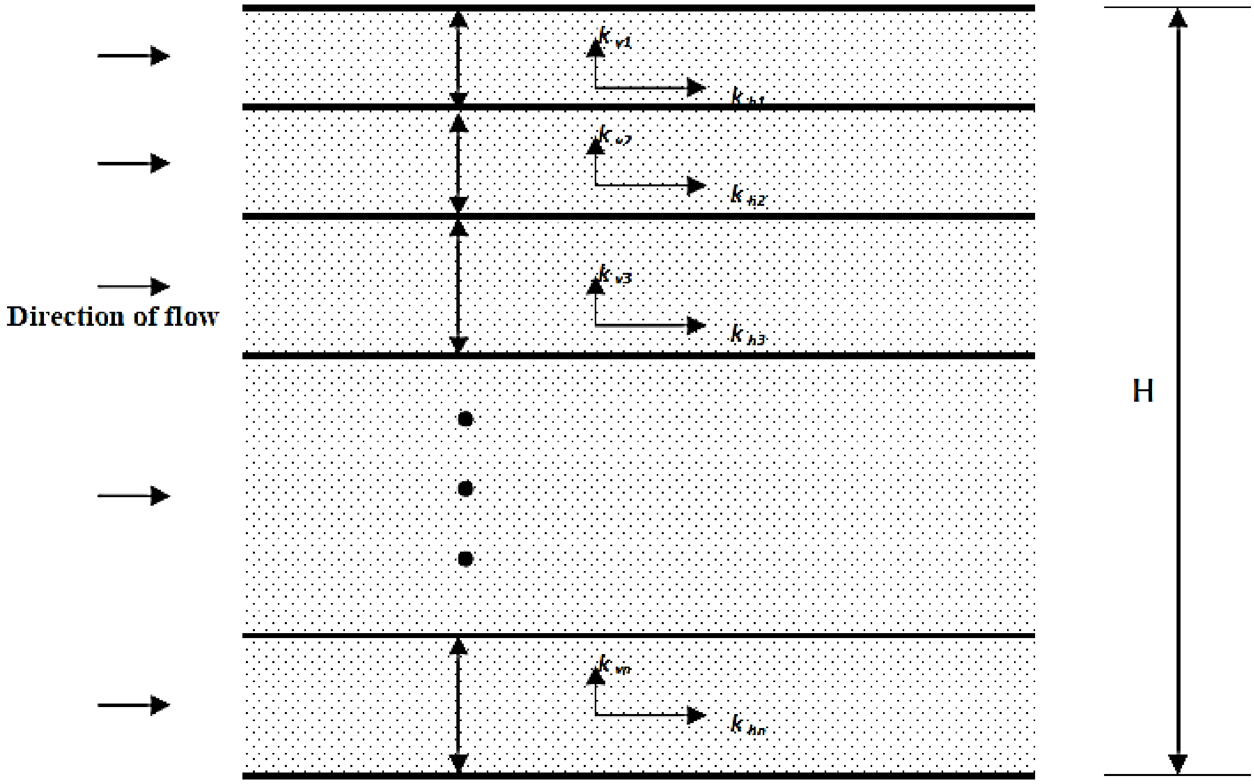
The velocity through soil I = v/n = 2/0.33 = 6 m/sec





5.7 Equivalent Permeability in Stratified Soil

5.7.1 Horizontal direction



$$q = v.1.H = v_1.1.H_1 + v_2.1.H_2 + v_3.1.H_3 + ... + v_n.1.H_n$$

Where ***v*** = average discharge velocity

v_1, v_2, v_3…v_n = discharge velocities of flow in layers denoted by the subscripts

From Darcy,s law

$$v = k_{H(eq)}.i_{eq}$$

$$v_1 = k_{h1}.i_1$$

$$v_1 = k_{h2}.i_2$$

$$v_1 = k_{h3}.i_3$$

$$\vdots$$

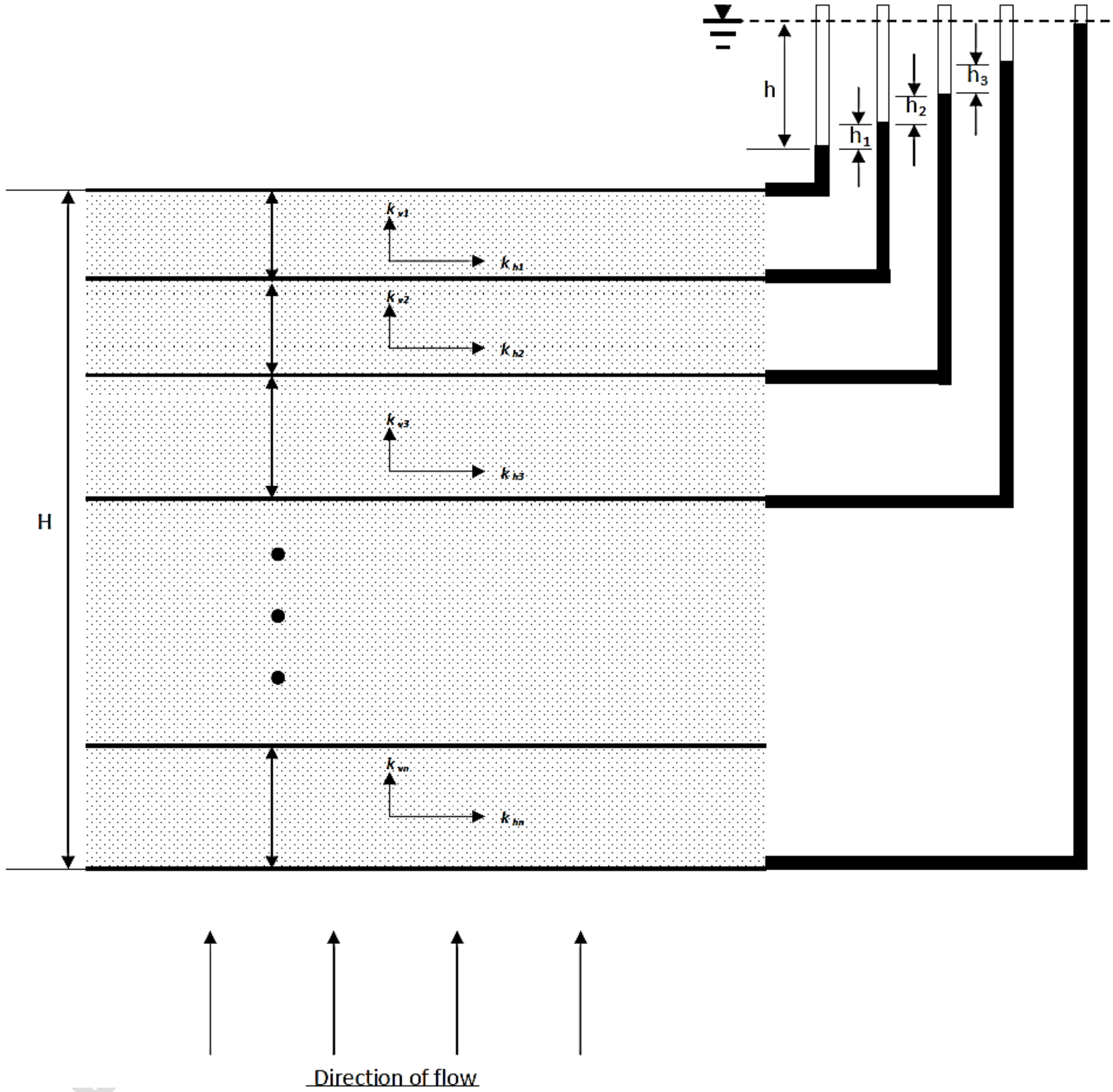
$$v_1 = k_{hn}.i_n$$

Since $i_{eq} = i_1 = i_2 = i_3 = i_n$ then

$$k_{H(eq)} = \frac{1}{H}\left(k_{h1}H_1 + k_{h2}H_2 + k_{h3}H_3 + \cdots + k_{hn}H_n\right)$$

$$k_{H(eq)} = \frac{\sum_{i=1}^{n} k_{hi} H_i}{H}$$

5.7.2 Vertical direction



$$v = v_1 = v_2 = v_3 = \cdots = v_n$$

and

$$h = h_1 + h_2 + h_3 + \cdots + h_n$$

Using Darcy's law $v = ki$, we can write

$$k_{v(eq)} \cdot \frac{h}{H} = k_{v1}.i_1 = k_{v2}.i_2 = k_{v3}.i_3 = \cdots = k_{vn}.i_n$$

$$h = H_1.i_1 + H_2.i_2 + H_3.i_3 + \cdots + H_n.i_n$$

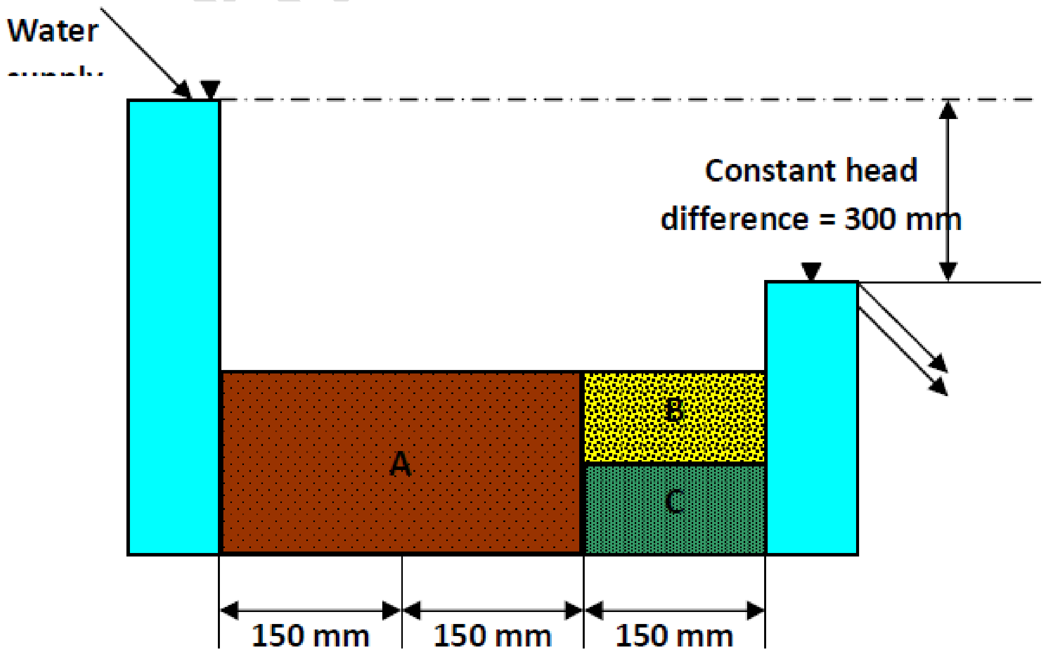
The solutions of these equations gives

$$k_{v(eq)} = \frac{H}{\left(\frac{H_1}{k_{v1}}\right) + \left(\frac{H_2}{k_{v2}}\right) + \left(\frac{H_3}{k_{v3}}\right) + \cdots + \left(\frac{H_n}{k_{vn}}\right)}$$

$$k_{v(eq)} = \frac{H}{\sum_{i=1}^{n} \frac{H_i}{k_{vi}}}$$

Example 5.11

The following figure shows the layers of soil in a tube 100mmx100mm in cross – section. Water is supplied to maintain a constant head difference of 300 mm across the sample. The permeability coefficient of the soils in the direction of flow through them are as follows: for soil A, $k_A = 1*10^{-2}$ cm/sec, soil B, $k_B = 3*10^{-3}$ cm/sec, soil C, $k_C = 5*10^{-4}$ cm/sec



Solution

For the soil layers B and C (the flow is parallel to the stratification)

$$k_{H(eq)} = \frac{1}{H}(k_{h1}H_1 + k_{h2}H_2) = \frac{1}{10}(3x10^{-3}(5) + 5x10^{-4}(5)) = 1.75x10^{-3} \quad \text{cm/sec}$$

For the layer A with equivalent layer of B and C

$$\therefore k_{eq} = \frac{H}{\frac{H_1}{k_1} + \frac{H_2}{k_2}} = \frac{45}{\frac{30}{1x10^{-2}} + \frac{15}{1.75x10^{-3}}} = 3.8x10^{-3}$$

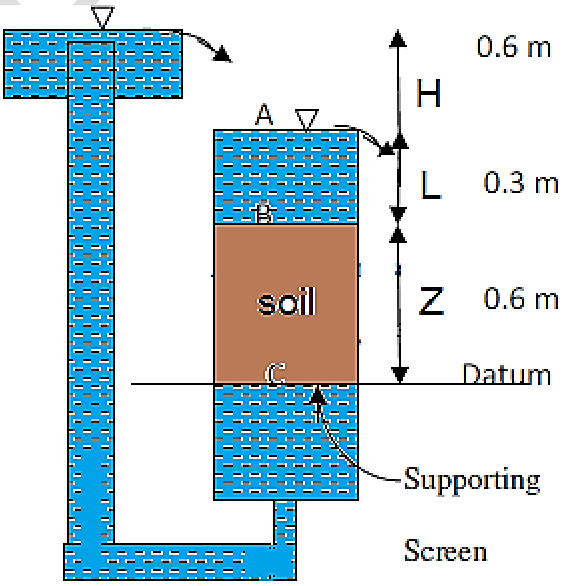
$$k_{eq} = 0.003888 cm/\sec$$

$$q = k_{eq} iA = 0.003888 \frac{300}{450}(10)^2 = 0.259\ cm^3/\sec$$

5.8 Seepage Force

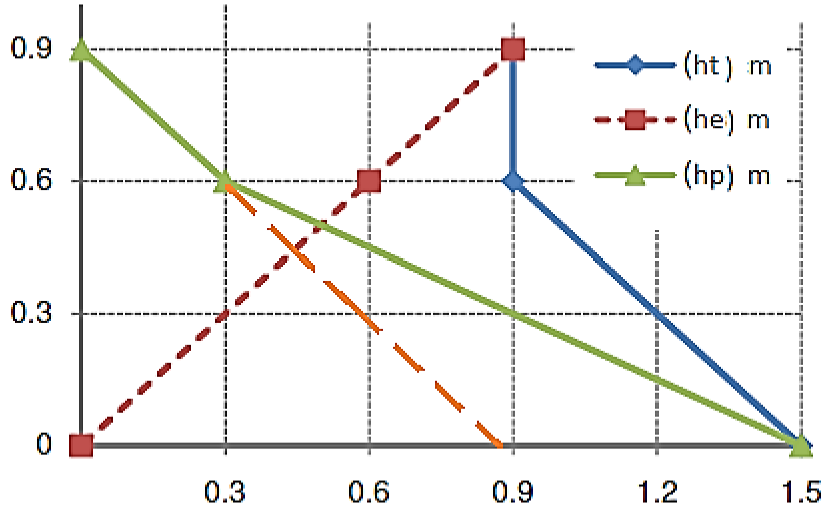
Example 5.12

For the setup shown draw h_t, h_e, h_p and find the seepage force. N =0.33. k = 0.5 cm/sec, γ_t = 20.9 kN/m^3



Solution

Point	h_e (m)	h_p (m)	h_t (m)
A	0.9	0	0.9
B	0.6	0.3	0.9
C	0	1.5	1.5

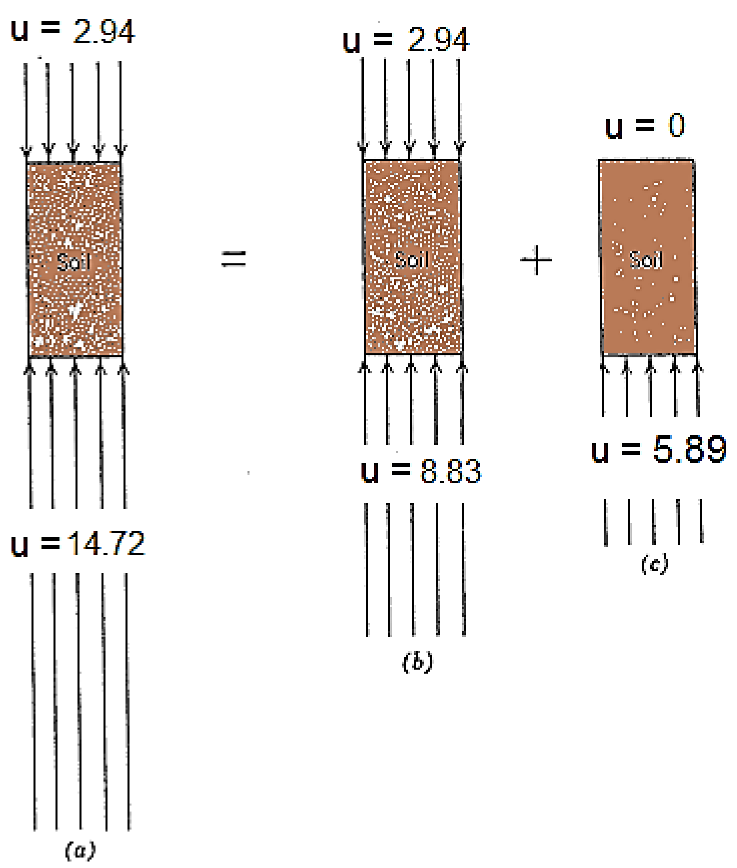


Elev. (m)	σ_v (kN/m^2)	u_s (kN/m^2)	$\sigma'_{v(s)}$ (kN/m^2)	u ($h_p\gamma_w$) (kN/m^2)	$\sigma'_{v(flow)}$ (kN/m^2)
0.9	0	0	0	0	0
0.6	9.81*0.3 = 2.94	9.81*0.3 = 2.94	0	9.81*0.3 = 2.94	0
0	2.94+20.9*0.6 = 15.48	9.81*0.9 = 8.83	**6.65**	1.5*9.81 = 14.72	**0.76**

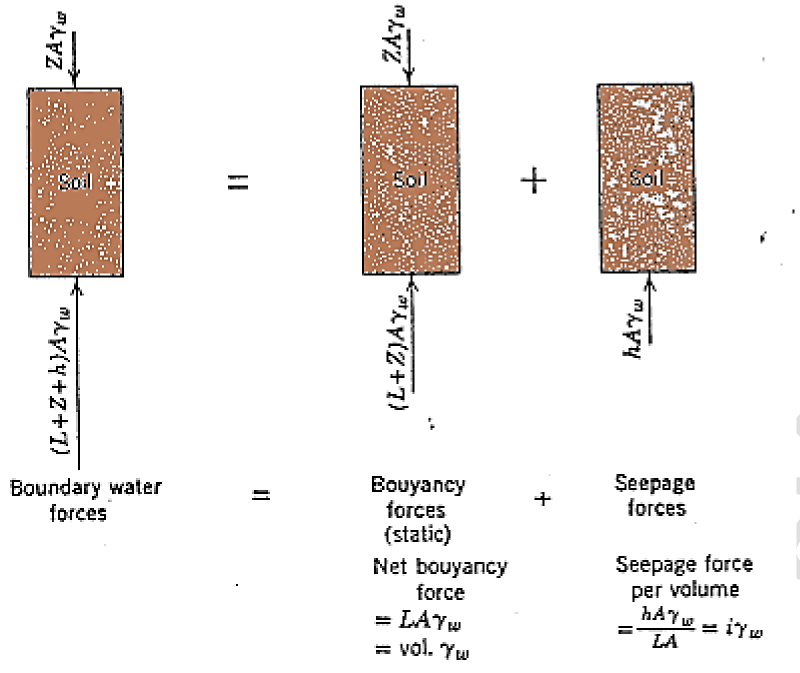
There is a great difference between the pore water pressure in soil when water at static state and in flow state

There is an upward water pressure on the bottom of the sample

The vertical pressure acts on the boundaries of soil sample (boundary water pressure)



(a) Boundary water pressure (b) buoyancy water pressure (c) pressure lost in seepage



Convert the pressure to force by multiplying them by area (A)

The magnitude of (a) and (b) depend on Z but in (c) it depends on (h)

(a) <u>Static situation</u>:

$\sigma_v = \gamma_w h_w + \gamma_{sat} z$

$u = \gamma_w (h_w + z)$

$\sigma_v' = \gamma' z$

(b) <u>Flow-Up Situation</u>:

$\sigma_v = \gamma_w h_w + \gamma_{sat} z$

$u = \gamma_w (h_w + z) + \mathbf{i\, z\, \gamma_w}$

$\sigma_v' = \gamma' z - \mathbf{i\, z\, \gamma_w}$

(c) <u>Flow-Down Situation</u>:

$\sigma_v = \gamma_w h_w + \gamma_{sat} z$

$u = \gamma_w (h_w + z) - \mathbf{i\, z\, \gamma_w}$

$\sigma_v' = \gamma' z + \mathbf{i\, z\, \gamma_w}$

The seepage force = $hA\gamma_w$

The force per unit volume $J = \frac{seepage\ force}{volume} = \frac{hA\gamma_w}{LA} = \frac{h}{L}\gamma_w = i\gamma_w$

Seepage forces usually act with the direction of flow.

<u>Quick Condition:</u>

The shear strength of cohesionless soil is directly proportional to the effective stress. When a cohesionless soil is subjected to a water condition that results in zero effective stress, the strength of the soil becomes zero and quick condition exists.

Quick condition: occurs in upward flow (for cohesionless soil) and when the total stress equals to pore water pressure.

$$\acute{\sigma}_{effect} = 0 = LA\gamma_w - hA\gamma_w = 0$$

$$\frac{h}{L} = i = \frac{\gamma_b}{\gamma_w} = i_c$$

i_c: The gradient required to cause a quick condition, termed critical gradient.

$$i_c = \frac{G-1}{1+e}$$

For equilibrium: downward force = upward force

γ_w (Z+h+L)*A = (γ_wZ+Lγ_t)*A

γ_wZ +γ_wh +γ_wL = γ_wZ+Lγ_t

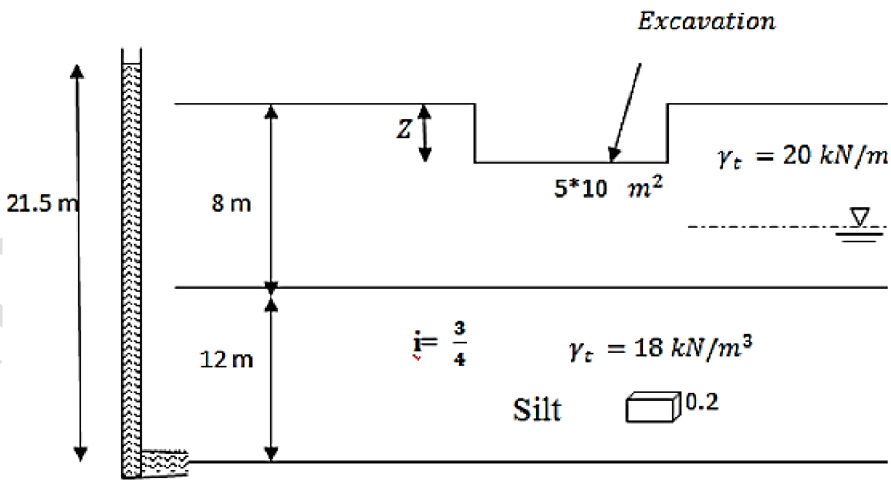
$$\frac{h}{L} = \frac{\gamma_t - \gamma_w}{\gamma_w} = \frac{\gamma_b}{\gamma_w} = i_c$$

$$\gamma_b = \frac{G-1}{1+e}\gamma_w, i_c = \frac{1}{\gamma_w}\frac{G-1}{1+e}\gamma_w = \frac{G-1}{1+e}$$

<u>Example 5.13</u>

Excavation is been carried out as shown in the figure. Find:

1. The depth Z that could cause boiling at the bottom of the clay layer.
2. The depth (Z) for the factor of safety against boiling equal to 2 at the bottom of the clay layer.
3. What is the thickness of the raft foundation that should be used before boiling occurs. If an uplift pressure of 60 kN/m^2 at the bottom of excavation exist ($\gamma_{concrete}$ = 25 kN/m^3).
4. Find the seepage force at an element of 0.2 m cube located at the center of silt layer.



<u>Solution</u>

$$ht_1 = 21.5$$

$$\text{1- } i = \frac{3}{4} = \frac{\Delta h_{1--2}}{10\ m} = \frac{21.5 - ht_2}{10} ------\therefore\ ht_2 = 14\ m$$

$$\therefore\ hp_2 = ht_2 - he_2 = 14 - 10 = 4\ m$$

To find Z

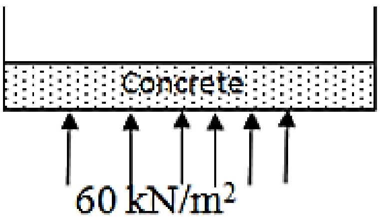
F. down = F. upward

A(8-z) *20 = 4.0 *10 *A ⟶ z= 6 m

2- $F.S = \frac{down\ ward\ force}{up\ ward\ force}$ $\qquad 2 = \frac{(8-z)*20*A}{4*10*A}$

$\therefore Z = 4\ m$

3- F down ward = F upward



$t*5*10*25 = 60*5*10$

$\therefore t = \frac{60}{25} = 2.4$ m (thickness of concrete)

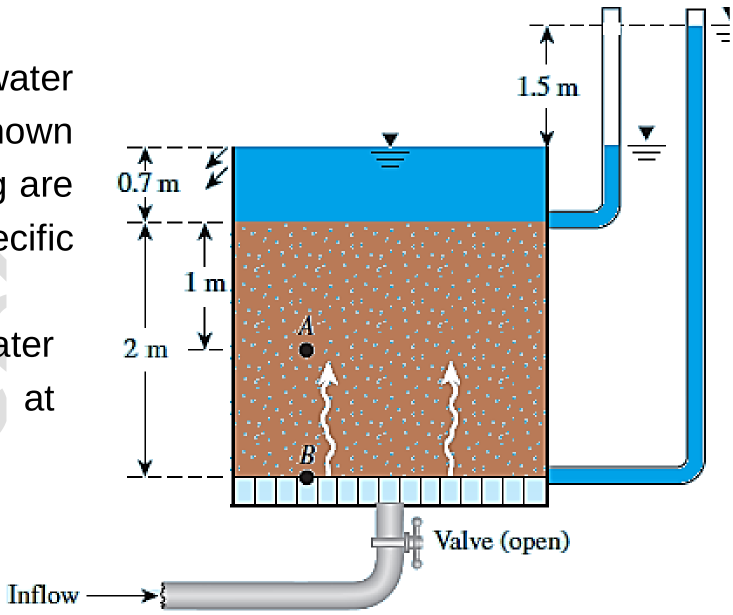
4- Seepage force = $i\gamma_w\ volume$ $= \frac{3}{4}*10*(0.2)^3 = 0.06$ kN

Example 5.14

Consider the upward flow of water through a layer of sand in a tank as shown in Figure. For the sand, the following are given: void ratio (e) = 0.52 and specific gravity of solids = 2.67.

a. Calculate the total stress, pore water pressure, and effective stress at points A and B.

b. What is the upward seepage force per unit volume of soil?



Solution

Part a: The saturated unit weight of sand is calculated as follows:

$$\gamma_{sat} = \frac{(G_s + e)\gamma_w}{1+e} = \frac{(2.67 + 0.52)9.81}{1 + 0.52} = 20.59\ \text{kN/m}^3$$

Point	Total stress, σ (kN/m²)	Pore water pressure, u (kN/m²)	Effective stress, $\sigma' = \sigma - u$ (kN/m²)
A	$0.7\gamma_w + 1\gamma_{sat} = (0.7)(9.81) + (1)(20.59) = \mathbf{27.46}$	$\left[(1+0.7) + \left(\frac{1.5}{2}\right)(1)\right]\gamma_w = (2.45)(9.81) = \mathbf{24.03}$	3.43
B	$0.7\gamma_w + 2\gamma_{sat} = (0.7)(9.81) + (2)(20.59) = \mathbf{48.05}$	$(2 + 0.7 + 1.5)\gamma_w = (4.2)(9.81) = \mathbf{41.2}$	6.85

Part b: Hydraulic gradient (i) = 1.5/2 = 0.75. The seepage force per unit volume can be calculated as $i\gamma_w = (0.75)*(9.81) = 7.36\ kN/m^3$

Summary of Main Points:

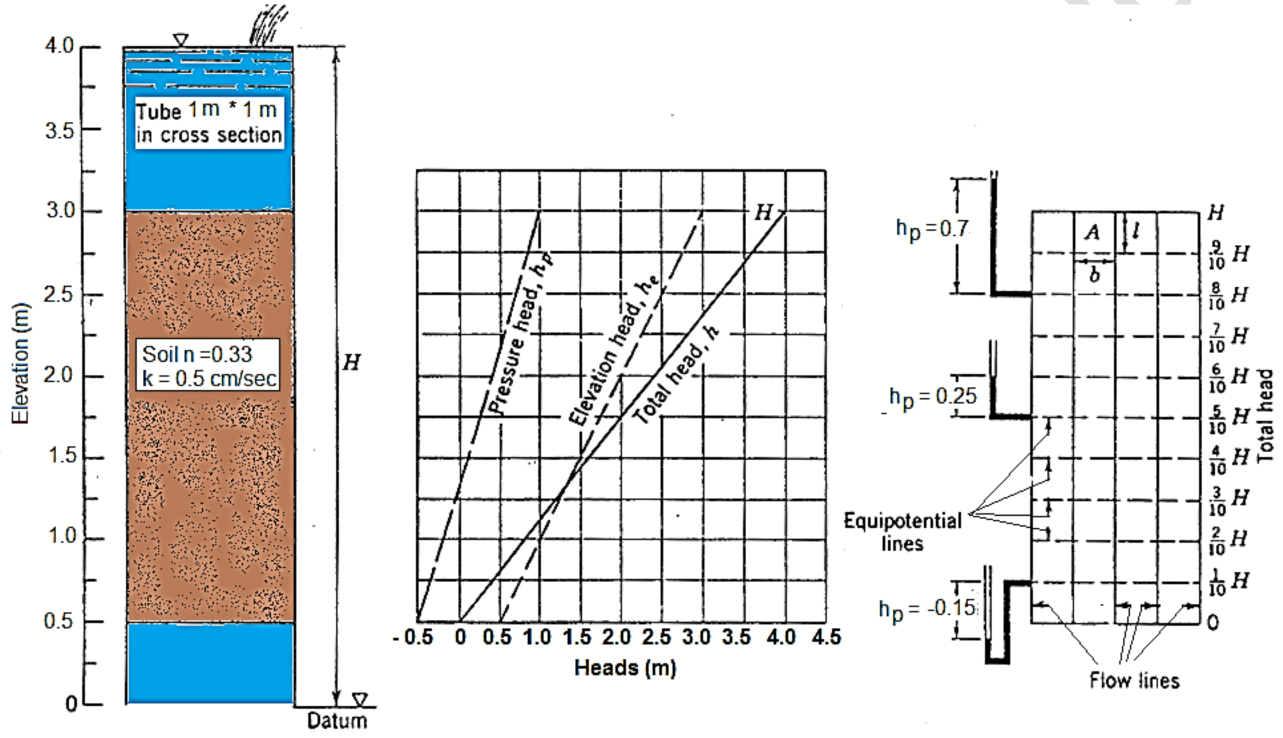
1- All total heads are lost in soil
2- Negative pore pressure can exist (the pore water pressure is less than atmospheric)
3- Direction of flow depends on the total head difference
4- Elevation head is an absolute magnitude depends on location of datum
5- The pressure magnitude is considered important since indicates the actual pressure in the water
6- the head is In soils v = ki
7- The seepage force per a volume of soil is $i*\gamma_w$ and acts in the direction of flow.
8- "Quick", refers to a condition wherein a cohesionless soil loses its strength because the upward flow of water makes the effective stress become zero.
9- When the flow is upward in the soil, pore water pressure increases and effective stress decreases. When the flow is downward, the pore water pressure decreases and the effective stress increases

5.9 Two Dimensional Flow

The figure shows a tube of 1.0*1.0m in cross section by 4.0m high. The values of total head, pressure head and elevation head are shown in the figure. The seepage is:

$$Q = kiA = 0.005 * \frac{4}{2.5} * (1 * 1) = 0.008 \text{ m}^3\text{/sec/m}$$

If we divided the soil by square mesh 0.25*0.25m as shown in the figure. Each horizontal line represents equal total heads (equipotential lines) and the vertical lines represent a flow line, the space between each flow line called flow chanal.



(a) (b) (c)

The flow through square A is:

$$q_A = ki_A A_A = k * \frac{\left(H - \frac{9}{10}H\right)}{l} * b$$

N_d = No. of drops in equipotential lines

$$i_A = \frac{H}{N_d l} \quad \rightarrow \quad q_A = k\frac{H}{N_d l} * b = k\frac{Hb}{N_d}$$

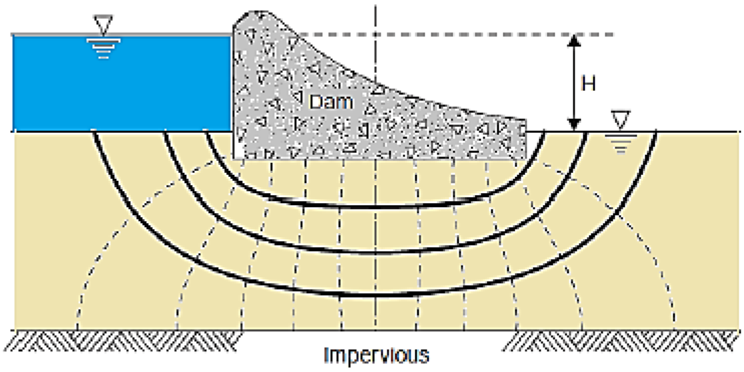
The quantity of seepage for all element below A is same (equal q)

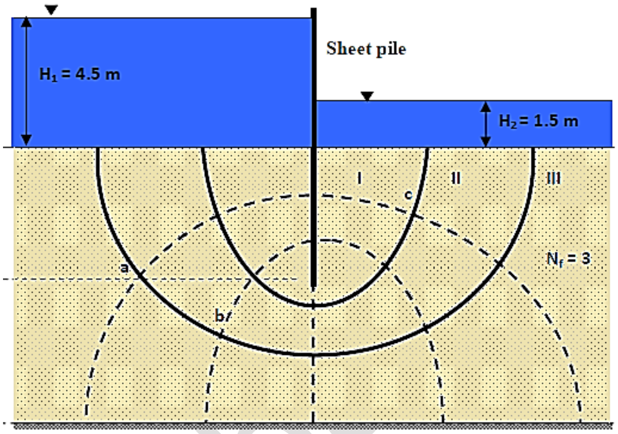
Total Q = qA * 4 * 1, Nf = 4 No. of flow channels

$$q = kH\frac{N_f}{N_d} = 0.005 * 4 * \frac{4}{10} = 0.008 \text{ m}^3\text{/sec/m}$$

∴ The problems of seepage can be solve using squares (flow net)

Most problems of flow are two-dimensional flows, e.g. are shown in Figure below:





The purpose of studying the flow in two Dimension are :

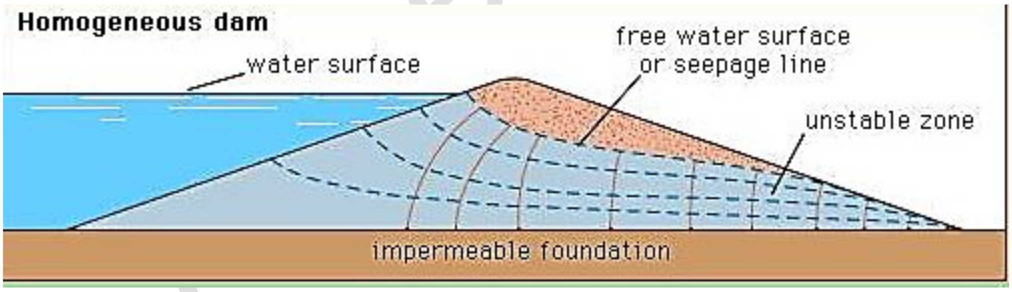
1- To find the amount of seepage per meter length (i.e. rate of flow).

2- Pressure distribution (pore water pressure)

3- Stability against piping or boiling.

4- Piezometer levels of selected point required.

To solve the problems of two-dimensional flow, Darcy's law is required to



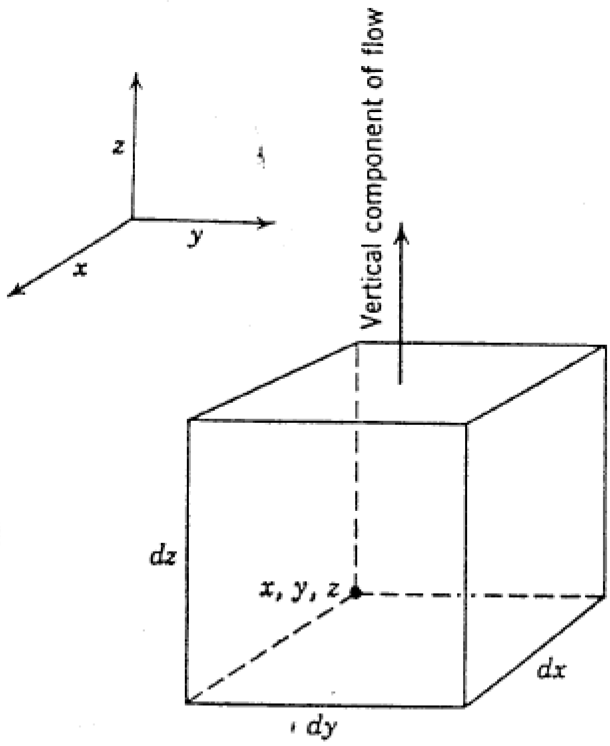
calculate the flow of water through the soil. In many instances, the flow of water through soil is not in one direction only. In such cases, the groundwater flow is generally calculated by the use of graphs referred to as flow nets. The concept of the flow net is based on Laplace's equation of continuity, which governs the steady flow condition for a given point in the soil mass.

5.10 Basic Equation for Two Dimensional Flow

Let us consider an element of soil as shown in figure, the flow is laminar with component in the *x*, *y*, and *z*-direction

$q = q_x + q_y + q_z$

Using Darcy's law, the following expressions for the vertical component of flow q_z:



Flow into the bottom of element

$q_z = kia$, where a is the area of bottom face

$$q_z = k_z\left(-\frac{\partial h}{\partial z}\right) dy\,dx$$

Flow out of top of element

$$q_z = \left(k_z + \frac{\partial k_z}{\partial z} dz\right)\left(-\frac{\partial h}{\partial z} - \frac{\partial^2 h}{\partial z^2} dz\right) dy\,dx$$

The net flow into the element from vertical flow = Δq_z = flow into bottom – flow out of top

$$\Delta q_z = k_z\left(-\frac{\partial h}{\partial z}\right) dy\,dx - \left(k_z + \frac{\partial k_z}{\partial z} dz\right)\times\left(-\frac{\partial h}{\partial z} - \frac{\partial^2 h}{\partial z^2} dz\right) dy\,dx$$

$$\Delta q_z = \left(k_z \frac{\partial^2 h}{\partial z^2} + \frac{\partial k_z}{\partial z}\frac{\partial h}{\partial z} + \frac{\partial k_z}{\partial z} dz \frac{\partial^2 h}{\partial z^2}\right) dx\,dy\,dz$$

For the condition of constant permeability

$$\Delta q_z = \left(k_z \frac{\partial^2 h}{\partial z^2}\right) dx\,dy\,dz$$

Similar the flow in *x*-direction

$$\Delta q_x = \left(k_x \frac{\partial^2 h}{\partial x^2}\right) dx\,dy\,dz$$

For the condition of two-dimensional flow $q_y = 0$

$$\Delta q = \Delta q_x + \Delta q_z = \left(k_z \frac{\partial^2 h}{\partial z^2} + k_x \frac{\partial^2 h}{\partial x^2}\right) dx\,dy\,dz$$

The volume of water in the element is

$$V_w = \frac{Se}{1+e}\,dx\,dy\,dz$$

And the rate of change of the water volume is equal to

$$\Delta q = \frac{\partial V_w}{\partial t} = \frac{\partial}{\partial t}\left(\frac{Se}{1+e}\,dx\,dy\,dz\right)$$

Since *dx dy dz (1+e)* = volume of solid in element and is constant

$$\Delta q = \frac{dx\,dy\,dz}{1+e}\frac{\partial(Se)}{\partial t}$$

Equating the two expressions for Δ*q*

$$\left(k_z\frac{\partial^2 h}{\partial z^2} + k_x\frac{\partial^2 h}{\partial x^2}\right)dx\,dy\,dz = \frac{dx\,dy\,dz}{1+e}\frac{\partial(Se)}{\partial t}$$

Which reduce to

$$k_z\frac{\partial^2 h}{\partial z^2} + k_x\frac{\partial^2 h}{\partial x^2} = \frac{1}{1+e}\left(e\frac{\partial S}{\partial t} + S\frac{\partial e}{\partial t}\right)$$

The above equation is the basic equation for two-dimensional laminar flow in soil. Looking at the *e* and *S* terms: four possible types of flow:

1. e and S both constant.
2. e varies and S constant.
3. e constant and S varies.
4. e and S both vary.

Type 1 is a steady flow, types 2, 3, and 4 are non-steady flow situations. Type 2 is consolidation for e decrease. Type 3 is constant-volume drainage for S decrease and imbibition for S increase. Type 4 includes compression and expansion problems. Types 3 and 4 are complex flow conditions for which satisfactory solutions have not been found.

For steady flow (e and S both constant) reduces to

$$k_z\frac{\partial^2 h}{\partial z^2} + k_x\frac{\partial^2 h}{\partial x^2} = 0$$

If $k_x = k_z$

$$\boxed{\frac{\partial^2 h}{\partial z^2} + \frac{\partial^2 h}{\partial x^2} = 0}$$

This is Laplace's equation. It's the basic equation of steady flow in isotropic soil. It means that the flow lines intersect the equipotential lines at right angle

Notes

1. The solution is represented by two families of curve (a) Equipotential lines and (b) Flowlines
2. The intersection of the two lines is represented by a flow net of square elements
3. Each element is a curvilinear square with dimension $\frac{b}{l} \cong 1$
4. The rate of flow is expressed /m length by $q = kH\frac{N_f}{N_d}$

   Where k: Coefficient of permeability

   H: Total head difference between the first and last equipotential lines

   $\frac{N_f}{N_d}$: Shape factor

Flowline is a line along which a water particle will travel from upstream to the downstream side in the permeable soil medium.

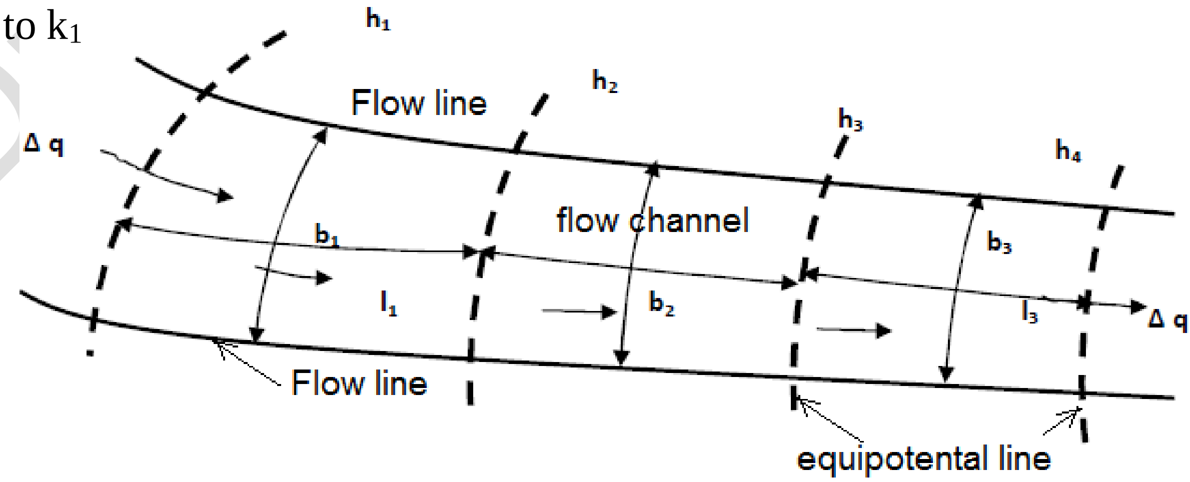
The equipotential line is a line along which the potential (total) head at all points is the same.

Flow channel: is the portion of flow net bounded by two adjacent flow lines

Flow net: the portion of approximate squares formed lines of flow lines and equipotential lines.

Impermeable (impervious) layer: A layer of material (such as clay) in an aquifer through which water does not pass or passes extremely slowly.

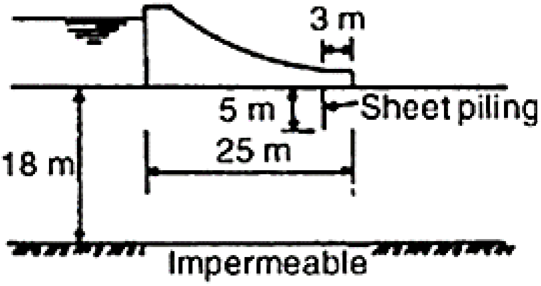
If $\frac{k_2}{k_1} \geq 10$ the soil of permeability k_2 is considered impermeable with respect to k_1



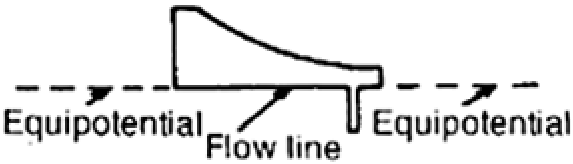
5.11 Steps for Two Dimensional Flow

1- Draw to scale the cross-section of the site of structure

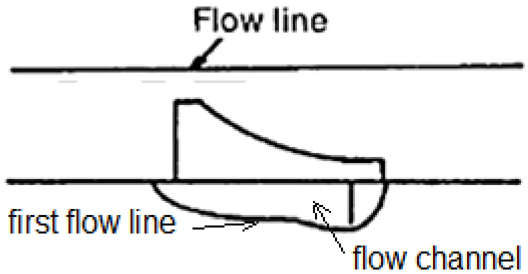
2- Draw the elevation of water at both sides of the structure and the impermeable layer.



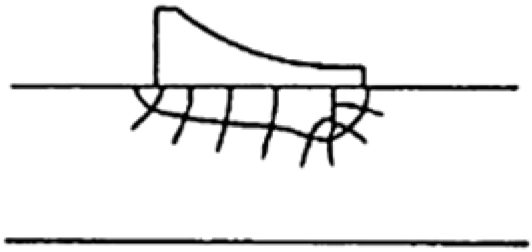
3- Identified the boundary condition



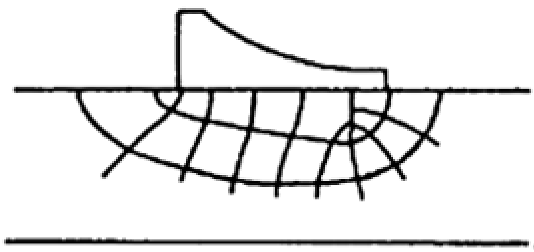
4- Draw the first flow line and established the first flow channel



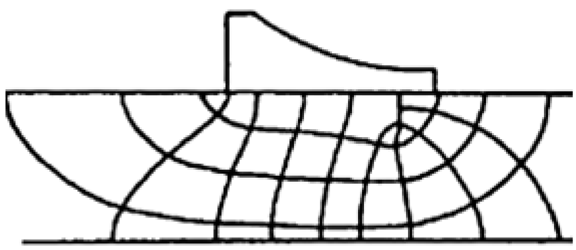
5- Divide the first flow channel into squares and determined the equipotential lines



6- Project the equipotential lines and start to draw the next flow line



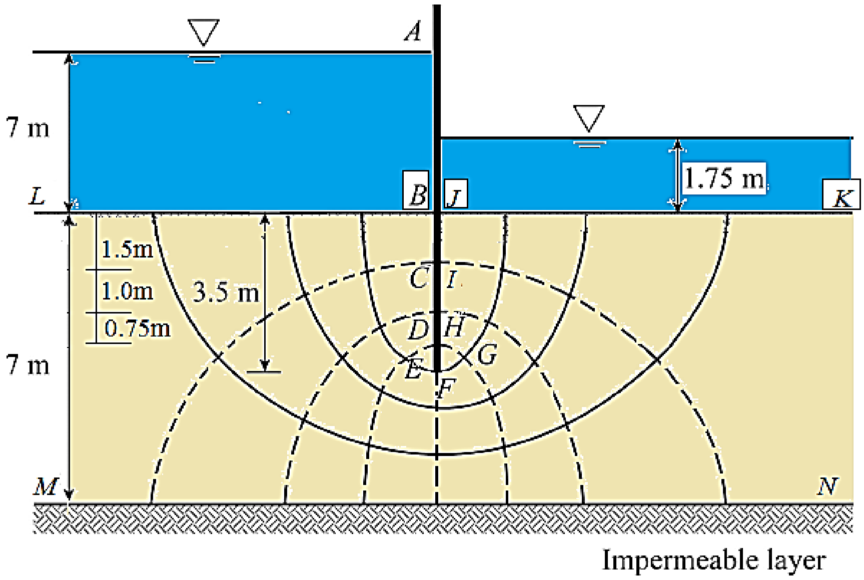
7- Project the equipotential lines and repeat until the end



Final flow net

Example 5.15

A sheet pile wall drove into silty soil having a permeability of 5*10^{-8} m/sec. find the quantity of seepage, pore water pressure at points (A to J), and exit gradient.



Solution

- The sheet pile wall length is in direction perpendicular to the page
- The problem is two-dimensional flow
- LB is upstream equipotential line
- JK is downstream equipotential line
- BF, FJ, and MN is flow lines
- H = 7 – 1.75 = 5.25 m, N_f = 4, N_d = 8

$q = kH\frac{N_f}{N_d} = 5 * 10^{-8} * 5.25 * \frac{4}{8} = 13.125 * 10^{-8}$ m^3/sec/m length

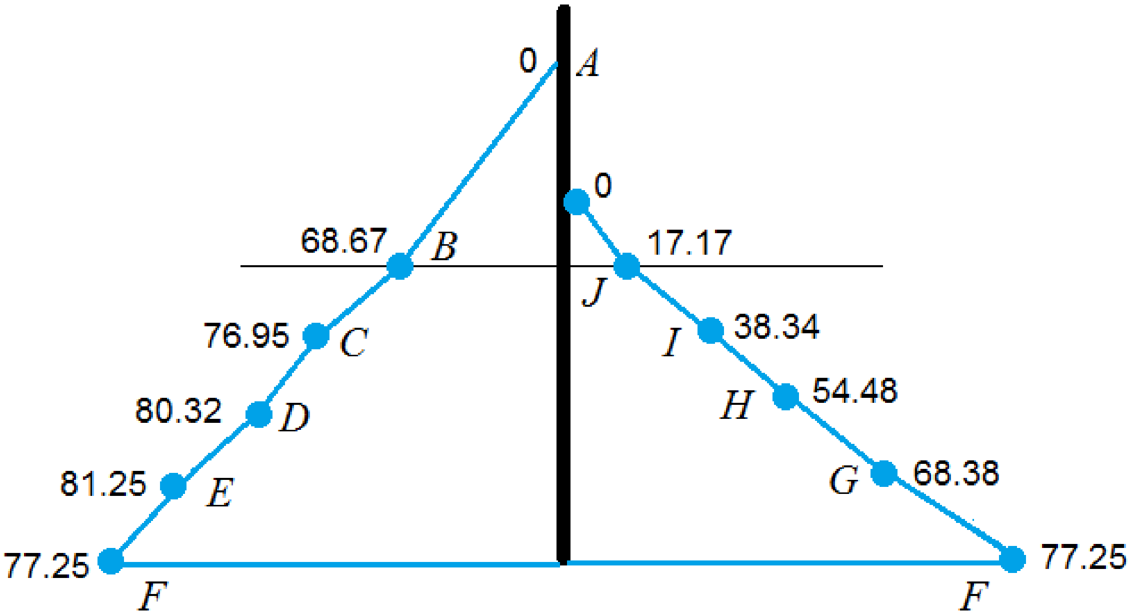
The difference in the total head is 5.25m and the N_d = 8 so in each drop, we loose 5.25/8 = 0.656m

Point	Ele. Head (m)	Total head (m)	Pressure head (m)	Pore water pressure (kN/m^2)
A	14	14.0	0	0
B	7	14	7.0	68.67
C	5.5	13.344	7.844	76.95
D	4.5	12.688	8.188	80.32
E	3.75	12.032	8.282	81.25
F	3.5	11.376	7.875	77.25
G	3.75	10.72	6.97	68.38
H	4.5	10.054	5.554	54.48
I	5.5	9.408	3.908	38.34
J	7.0	8.75	1.75	17.17

End	8.75	8.75	0	0

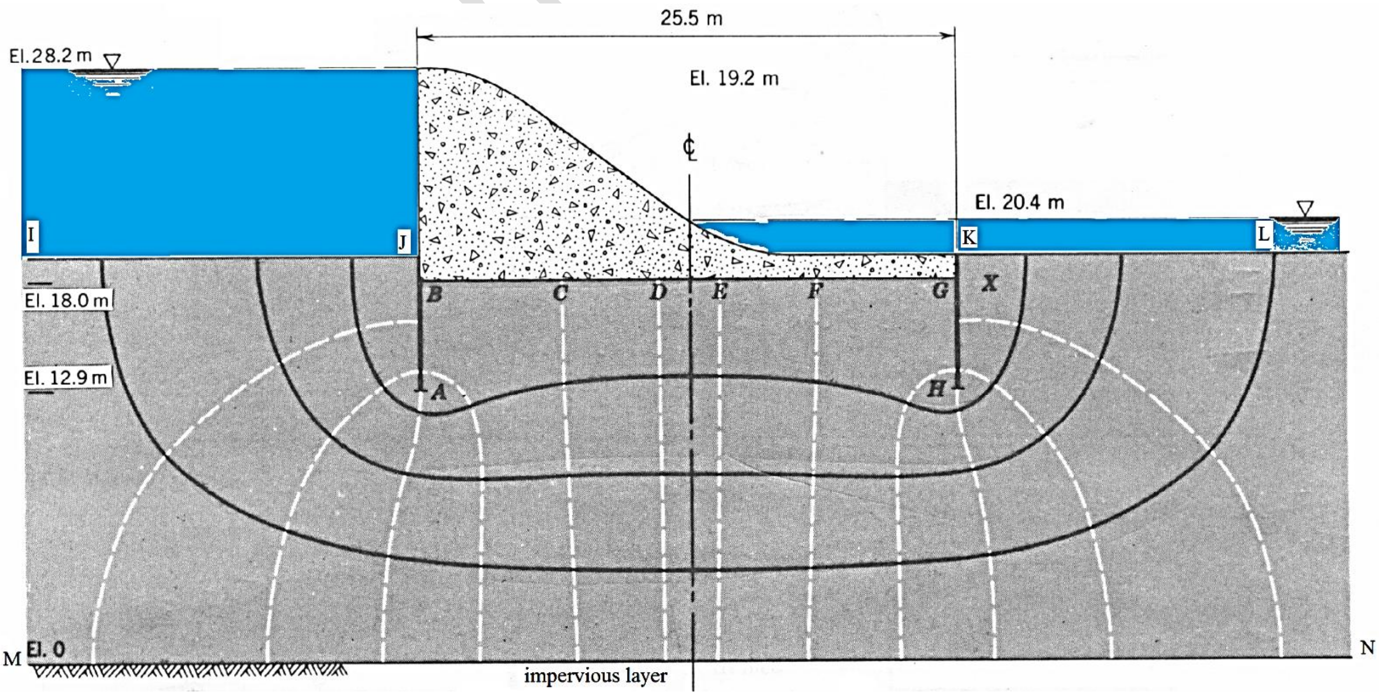
The exit gradient is found at the downstream, it locates in the square near the sheet pile. If the dimension of this square is 1.5m

$$i_e = \frac{\Delta h}{l} = \frac{0.656}{1.5} = 0.437 \; < 1.0 \; ok$$



Example 5.16

Find the pressure head acts at points (A to H). What is the quantity of seepage in point X for the concrete dam shown if k = 0.05 * cm/sec.



- The concrete dam length is in direction perpendicular to the page
- The problem is two-dimensional flow
- IJ is upstream equipotential line
- KL is downstream equipotential line
- IJ, KL, JABGHK, and MN is flow lines
- H = 28.2 – 20.4 = 7.8 m.
- N_f = 4, in this figure and due to symmetry around y-axis of the flow net the middle square is not complete and its equal approximately to 0.6 so that N_d = 12.6

$q = kH\frac{N_f}{N_d} = 0.05 * 10^{-2} * 7.8 * \frac{4}{12.6} = 0.00124$ m³/sec/m length

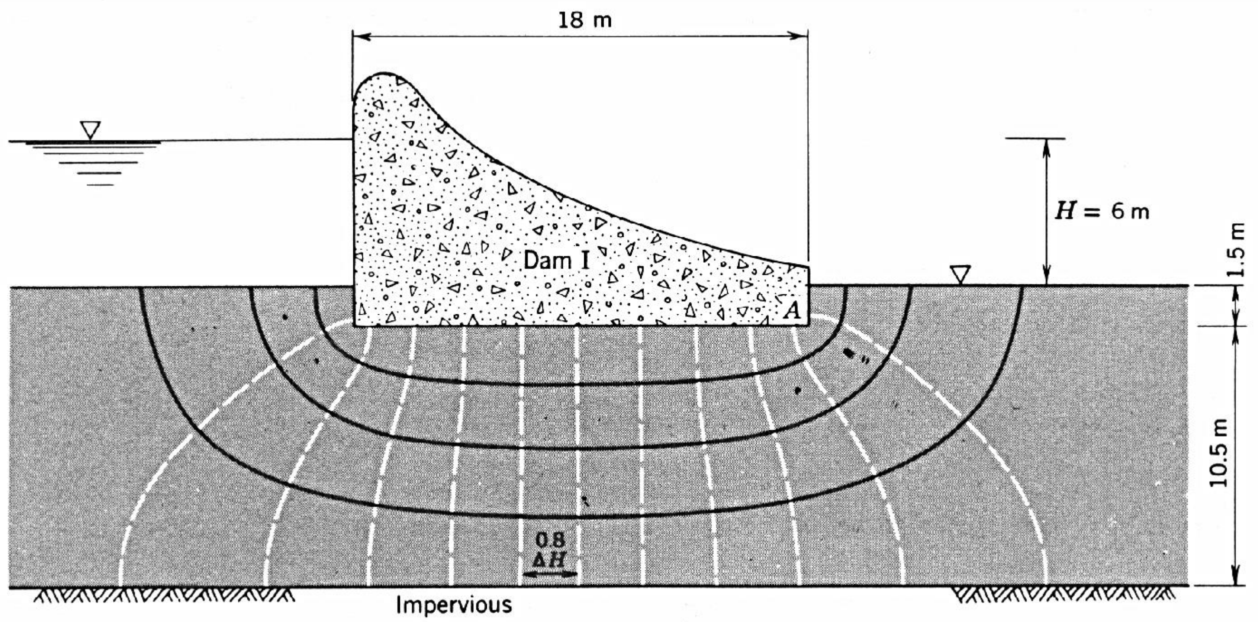
The difference in the total head is 7.8m and the N_d = 12.6 so in each drop, we loose 7.8/12.6 = 0.62m

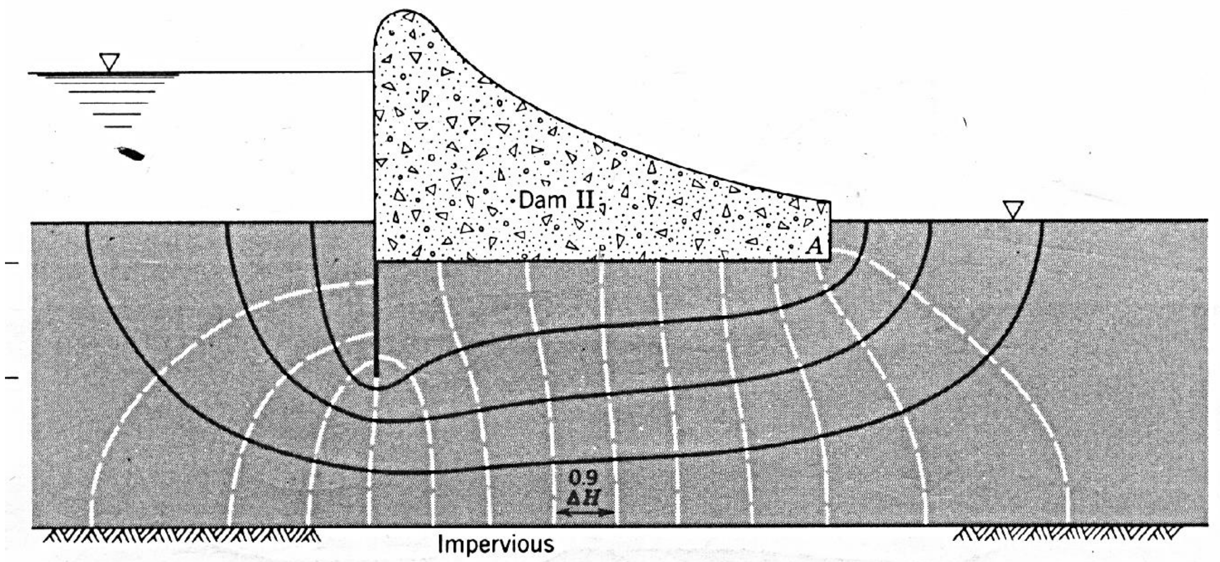
Point	Ele. Head (m)	Total head (m)	Pressure head (m)
A	12.9	26.34	13.44
B	18	25.41	7.41
C	18	25.10	7.1
D	18	24.48	6.48
E	18	24.11	6.11
F	18	23.49	5.49
G	18	23.18	5.18
H	12.9	22.25	9.35

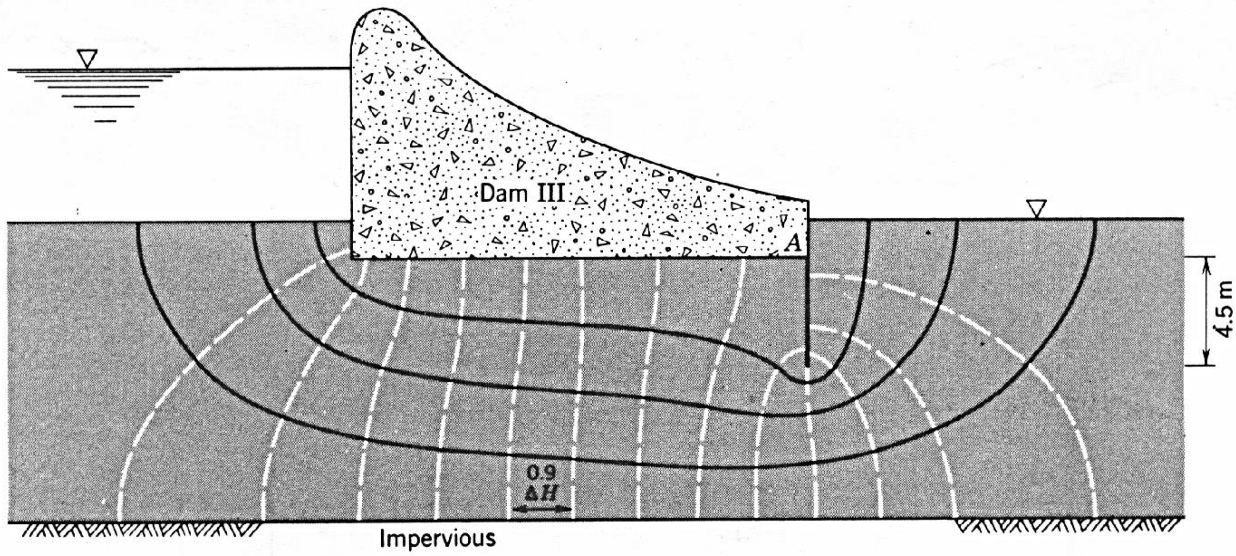
$$i_e = \frac{\Delta h}{l} = \frac{0.62}{3.3} = 0.19 < 1.0\ ok$$

Example 5.17

For the dams shown determine the quantity of seepage, uplift pressure at point A and exit gradient, if k = $5*10^{-6}$ m/sec.


18 m
$H = 6$ m
1.5 m
Dam I
A
10.5 m
0.8
ΔH
Impervious


Dam II
A
0.9
ΔH
Impervious


Dam III
A
4.5 m
0.9
ΔH
Impervious

Solution

Dam	N_f	N_d	q (m^3/sec/m)	Uplift pressure at A (kN/m^2)	Exit gradient
I	4	12	10.29	2.25	0.42
II	4	14	8.82	2.13	0.34
III	4	14	8.82	2.87	0.18

The idea of this example is to find the difference when adding sheet pile at upstream or downstream.

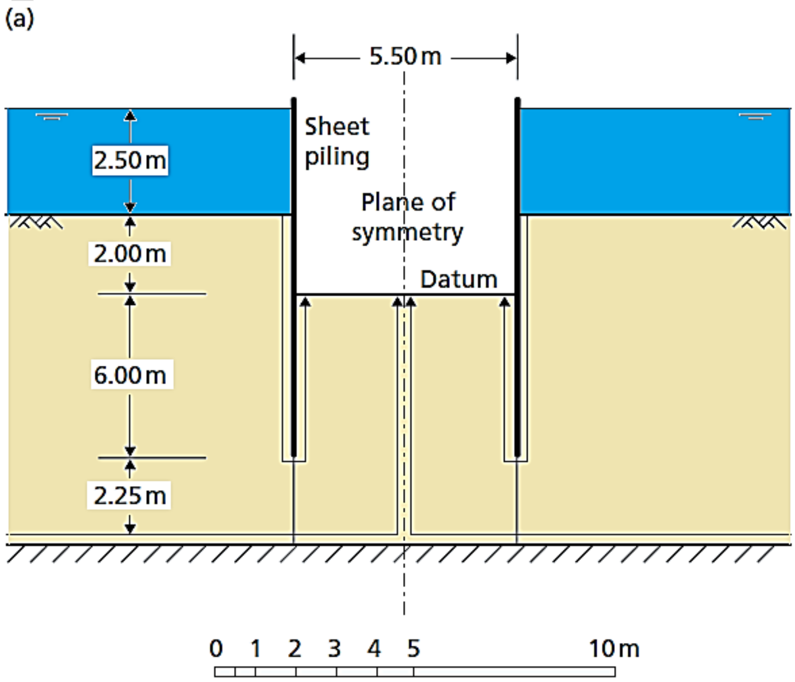
- If the sheet pile at upstream, minimum uplift pressure can be get
- If the sheet pile at downstream, minimum exit gradient can be get
- The worst case without sheet pile

i_{exit} can be reduced using the following

- Placing sheet pile beneath the dam on either side
- Using sheet pile on both sides
- Increase length of sheet pile
- Increase length of dam
- Applied the crash rock on downstream side of the dam

Example 5.18

A riverbed consists of a layer of sand overlying impermeable rock; the depth of water is 2.50 m. A long cofferdam 5.50 m wide is formed by driving two lines of sheet piling to a depth of 6.00 m below the level of the river bed, and excavation to a depth of 2.00 m below bed level is carried out within the cofferdam. The water level within the cofferdam is kept at excavation level by pumping. If the flow of water into the cofferdam is 0.25 m^3/h per unit length, what is



the coefficient of permeability of the sand? What is the hydraulic gradient immediately below the excavated surface?

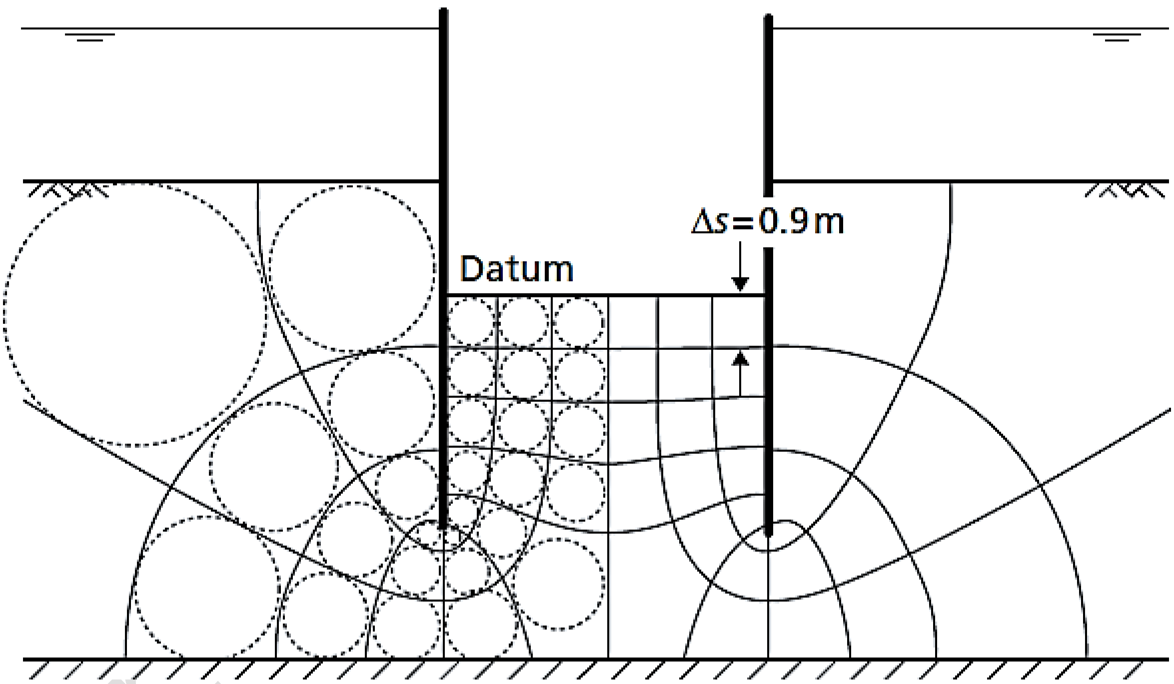
Solution

The section and boundary conditions appear in Figure (a) and the flow net is shown in Figure (b). In the flow net, there are six flow channels (three per side) and ten equipotential drops. The total head loss is 4.50 m. The coefficient of permeability is given by

$$k = \frac{q}{h(N_f / N_d)}$$

$$= \frac{0.25}{4.50 \times 6 / 10 \times 60^2} = 2.6 \times 10^{-5}\ \text{m/s}$$

(b)



The distance (Δs) between the last two equipotentials is measured as 0.9 m. The required hydraulic gradient is given by

$$i = \frac{\Delta h}{\Delta s}$$

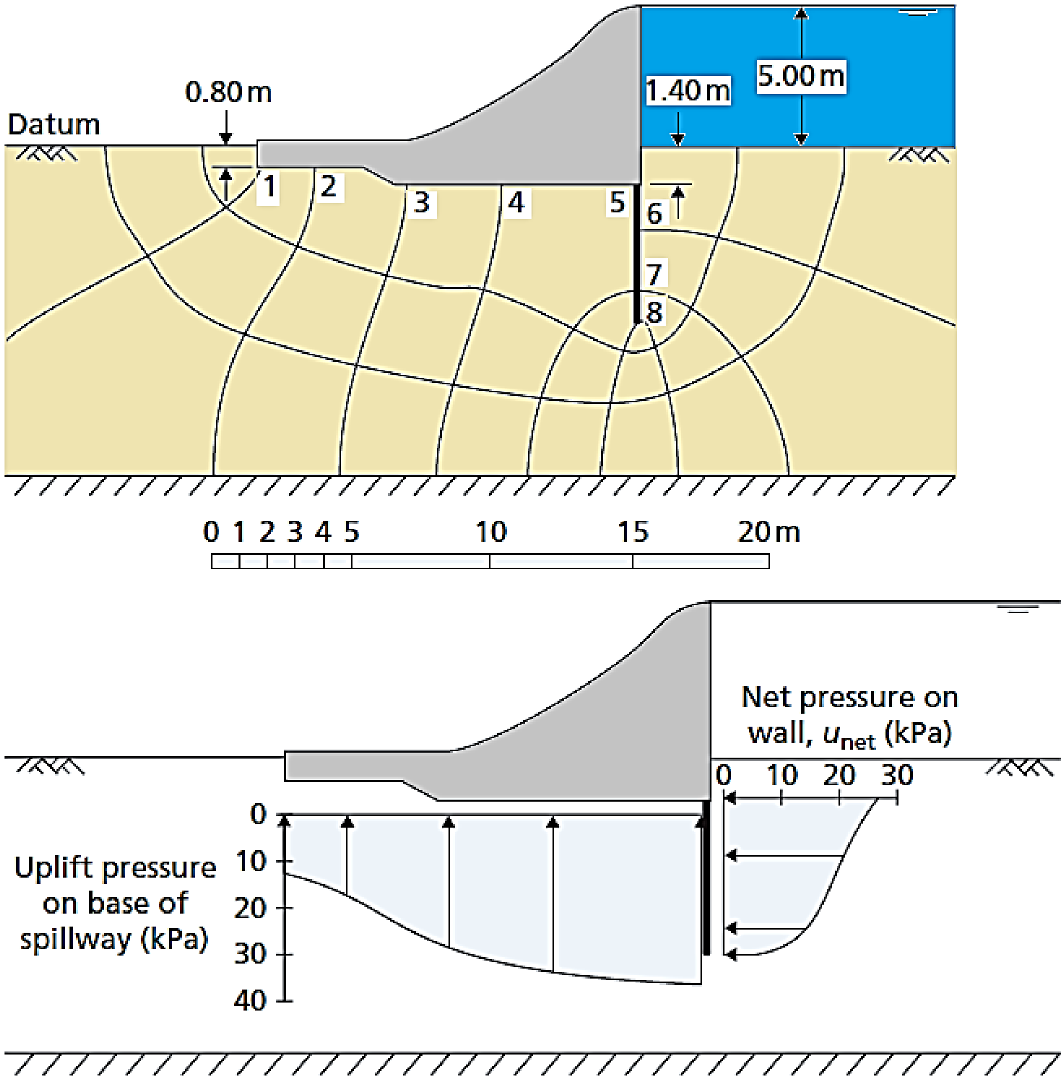
$$= \frac{4.50}{10 \times 0.9} = 0.50$$

Example 5.19

The section through a dam spillway is shown in Figure. Determine the quantity of seepage under the dam and plot the distributions of uplift pressure on the base of the dam, and the net distribution of water pressure on the cut-off wall at the upstream end of the spillway. The coefficient of permeability of the foundation soil is 2.5 × 10^{-5} m/s.

Solution

The flow net is shown in Figure. The downstream water level (ground surface) is selected as a datum. Between the upstream and downstream equipotential, the total head loss is 5.00 m. In the flow net, there are three flow channels and ten equipotential drops. The seepage is given by



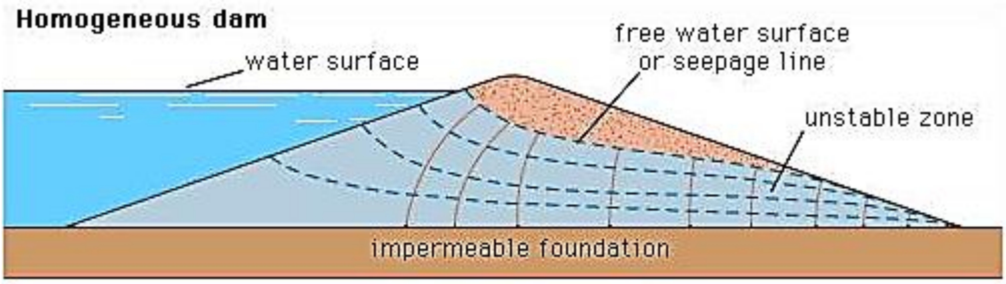
The calculations are shown in Table, and the pressure diagram is plotted in Figure. The levels (z) of points 5–8 in Table were found by scaling from the diagram.

Point	h(m)	z(m)	h – z(m)	$u = \gamma_w(h - z)$ (kPa)
1	0.50	–0.80	1.30	12.8
2	1.00	–0.80	1.80	17.7
3	1.50	–1.40	2.90	28.4
4	2.00	–1.40	3.40	33.4
5	2.30	–1.40	3.70	36.3

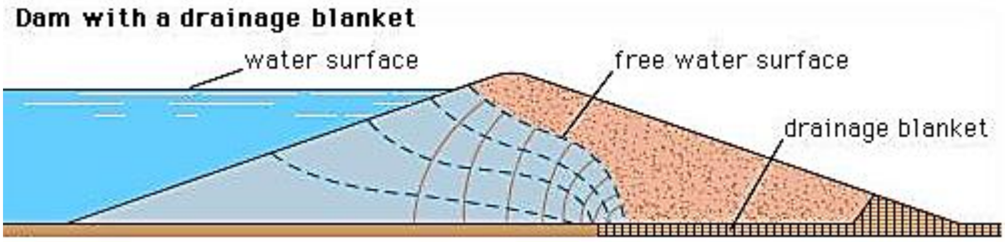
Level	z(m)	h_b (m)	u_b/γ_w (m)	h_f (m)	u_f/γ_w (m)	$u_b - u_f$ (kPa)
5	–1.40	5.00	6.40	2.28	3.68	26.7
6	–3.07	4.50	7.57	2.37	5.44	20.9
7	–5.20	4.00	9.20	2.50	7.70	14.7
8	–6.00	3.50	9.50	3.00	9.00	4.9

5.12 Seepage through Earth Dam

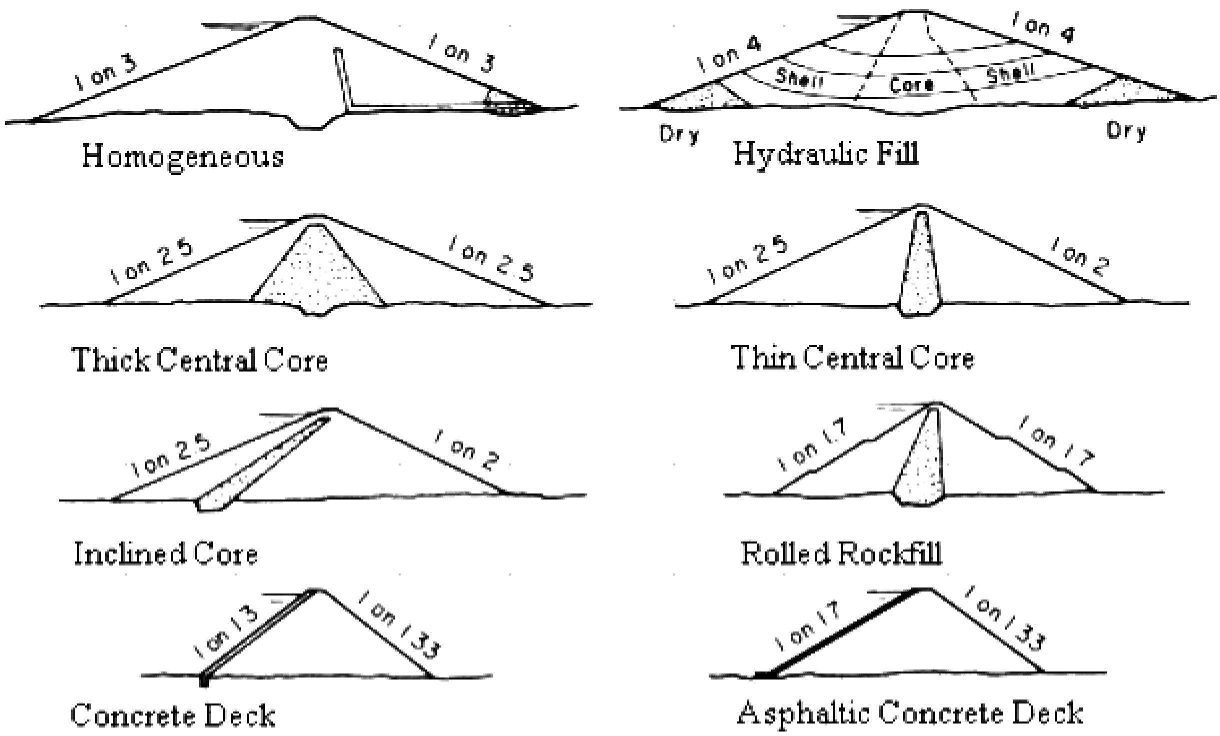
Earth dams are used to store a large amount of water since the dam is constructed using soil, thus there is a flow through this soil



From the figure above it can be seen that the downstream of the dam is unstable zone, thus always used drain must be used.

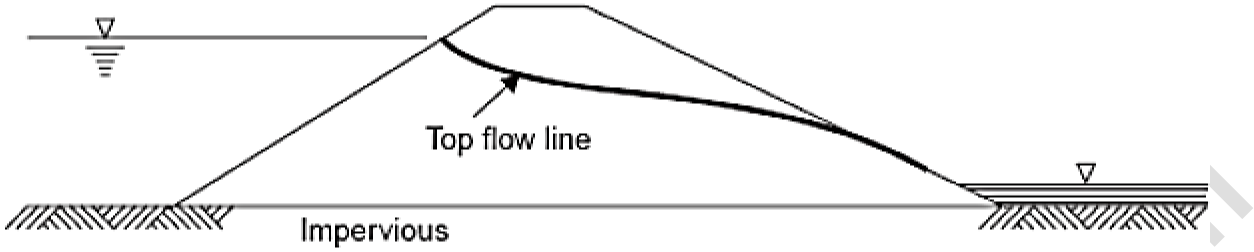


Several types of earth dam are found

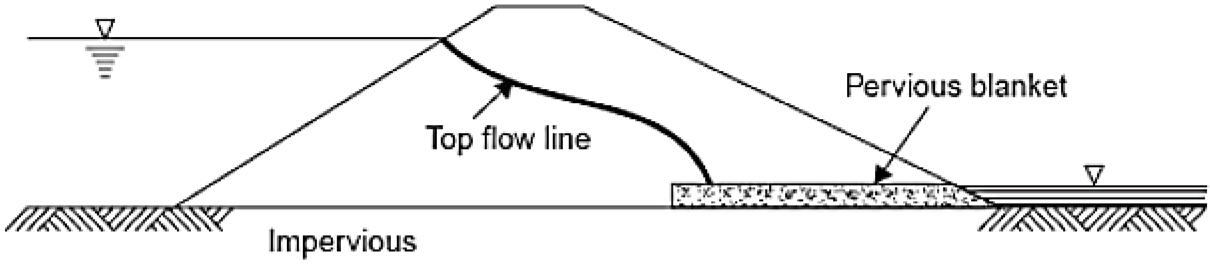


The flow through an earth dam differs from the other cases in that the top flow line is not known in advance of sketching the flow net. Thus, it is a case of

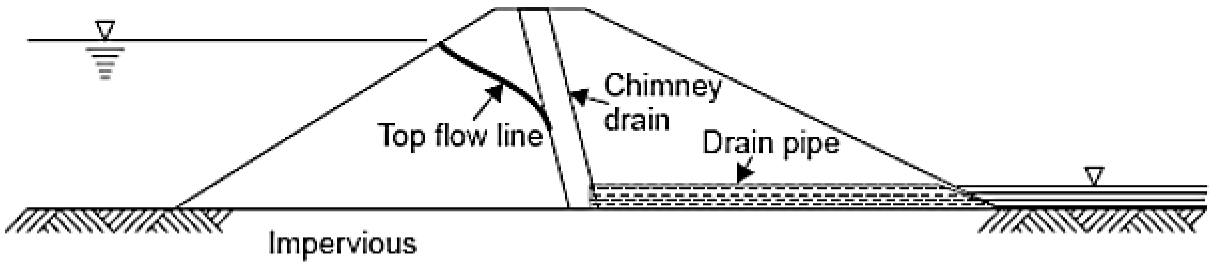
unconfined flow. The top flow line, as well as the flow net, will be dependent upon the nature of internal drainage for the earth dam. Typical cases are shown in Figure; the top flow line only is shown.



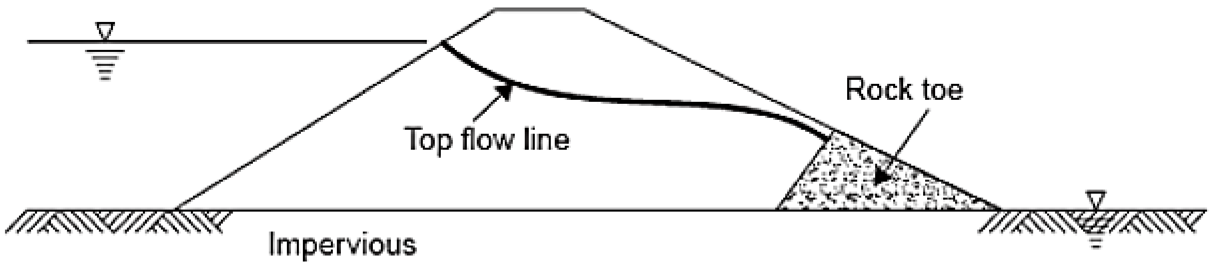
(a) Homogeneous dam without internal drain



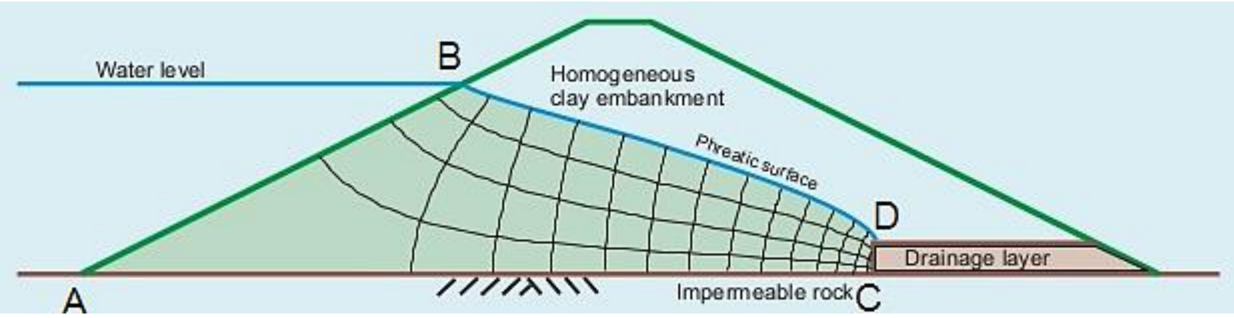
(b) Homogeneous dam with underdrain on pervious blanket



(c) Homogeneous dam with chimney drain



(d) Homogeneous dam with rock toe on the downstream side



Boundary condition

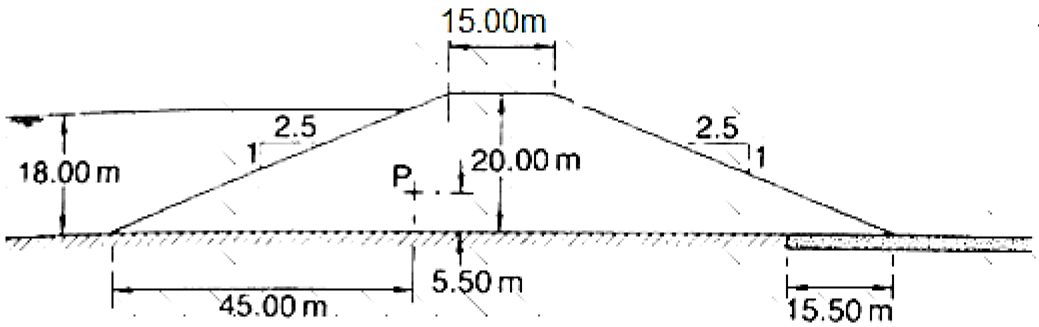
AB is equipotential line

AC is flow line

BD is the top flow line, at all points of this line the pressure head is zero. Thus BD is also the 'phreatic line'; or, on this line, the total head is equal to the elevation head

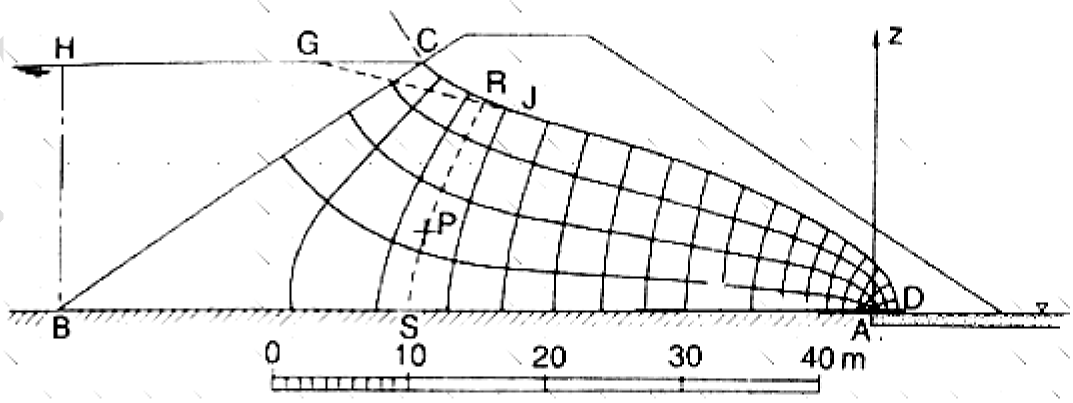
Example 5.20

For the earth dam shown draw the flow net and find the quantity of seepage, if k $5*10^{-8}$ m/sec



(a)

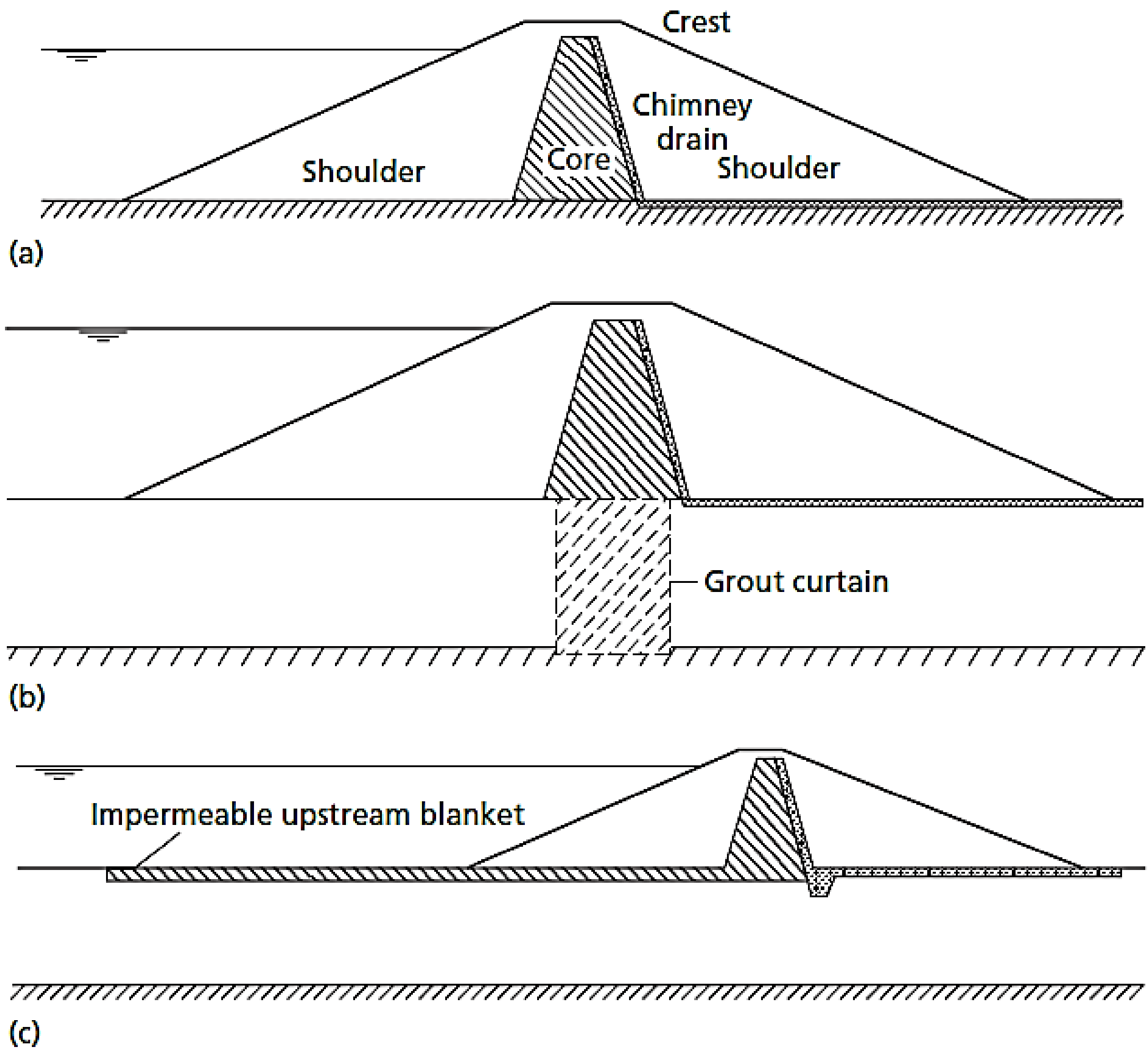
Solution



$q = kH\frac{N_f}{N_d} = 5 * 10^{-8} * 18 * \frac{4}{18} = 2 * 10^{-7}$ m³/sec/m

5.13 Seepage Control in Embankment Dams

The design of an embankment dam section and, where possible, the choice of soils are aimed at reducing the effects of seeping water. Where high hydraulic gradients exist there is a possibility that the seeping water may cause internal erosion within the dam, especially if the soil is poorly compacted. Erosion can work its way back into the embankment. A section with a central core of low permeability, aimed at reducing the volume of seepage, is shown in Figure.



5.14 The Design of Filter

Filters used to control seepage must satisfy certain fundamental requirements.

- The pores must be small enough to prevent particles from being carried in from the adjacent soil.
- The permeability must simultaneously be high enough to ensure the free drainage of water entering the filter.
- The capacity of a filter should be such that it does not become fully saturated.
- In the case of an embankment dam, a filter placed downstream from the core should be capable of controlling and sealing any leak which develops through the core as a result of internal erosion.
- The filter must also remain stable under the abnormally high hydraulic gradient which is liable to develop adjacent to such a leak.

It has been shown that filter performance can be related to the size D_{15} obtained from the particle size distribution curve of the filter material. The characteristics of the adjacent soil, in respect of its retention by the filter, can be represented by the size D_{85} for that soil. The following criterion has been recommended for satisfactory filter performance:

$$\frac{(D_{15})_f}{(D_{85})_s} < 5$$

Where (D15)f and (D85)s refer to the filter and the adjacent (upstream) soil, respectively.

In the case of filters for fine soils the following limit is recommended for the filter material:

$$D_{15} \leq 0.5\,\text{mm}$$

5.15 Flow Nets in Anisotropic Medium

In nature, most soils exhibit some degree of anisotropy. So to account for soil anisotropy with respect to permeability, some modification of the flow net construction is necessary, where most soils are homogenous anisotropy.

Homogenous soil: soils that do not vary in properties in both direction from point to point vertically or horizontally.

Isotropic soil: soils that have similar properties at a given location at al planes in all directions

In anisotropic soils, the differential equation of continuity for two – dimensional flow in, where $k_x \neq k_z$, is

$$k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0$$

In that case, the equation represents two families of curves that do not meet at 90°. However, we can rewrite the preceding equation as

$$\frac{\partial^2 h}{(k_z / k_x)\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0$$

Substituting $x' = \sqrt{k_z / k_x}.x$ then

$$\boxed{\frac{\partial^2 h}{\partial x'^2} + \frac{\partial^2 h}{\partial z^2} = 0}$$

To construct the flow net, use the following procedures:

1. Adopt a vertical scale (that is, z-axis) for drawing the cross-section.
2. Adopt a horizontal scale (that is, x-axis) such that horizontal scale $=\sqrt{k_z/k_x}$ (vertical scale).
3. With scales adopted in steps 1 and 2, plot the vertical section through the permeable layer parallel to the direction of flow.
4. Draw the flow net for the permeable layer on the section obtained from step 3, with flow lines intersecting equipotential lines at right angles and the elements as approximate squares.

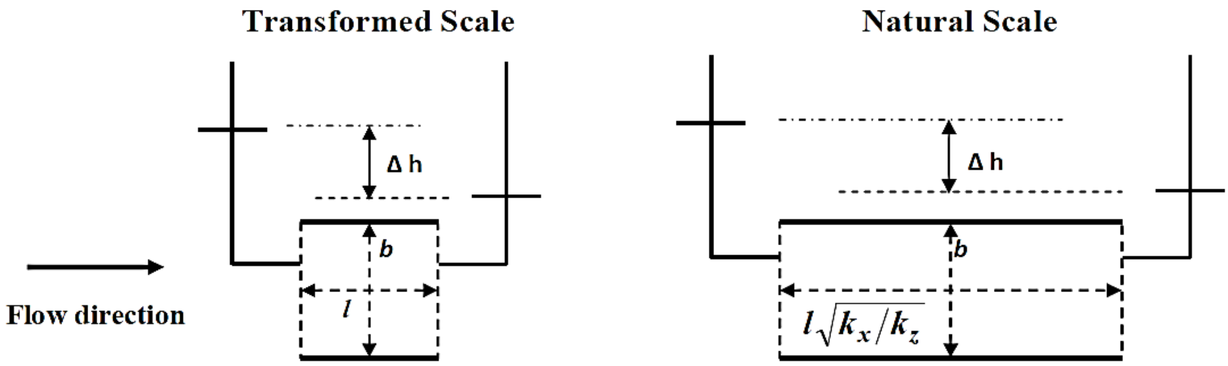
Depending on the problem geometry, we can also adopt transformation in the z-axis direction, in the same manner, describe above by adopting horizontal scale and the vertical scale will equal horizontal scale multiplying by $\sqrt{k_x/k_z}$ i.e. that the continuity equation will be written as follow:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z'^2} = 0 \quad \text{where } z' = \sqrt{k_x/k_z}.z$$

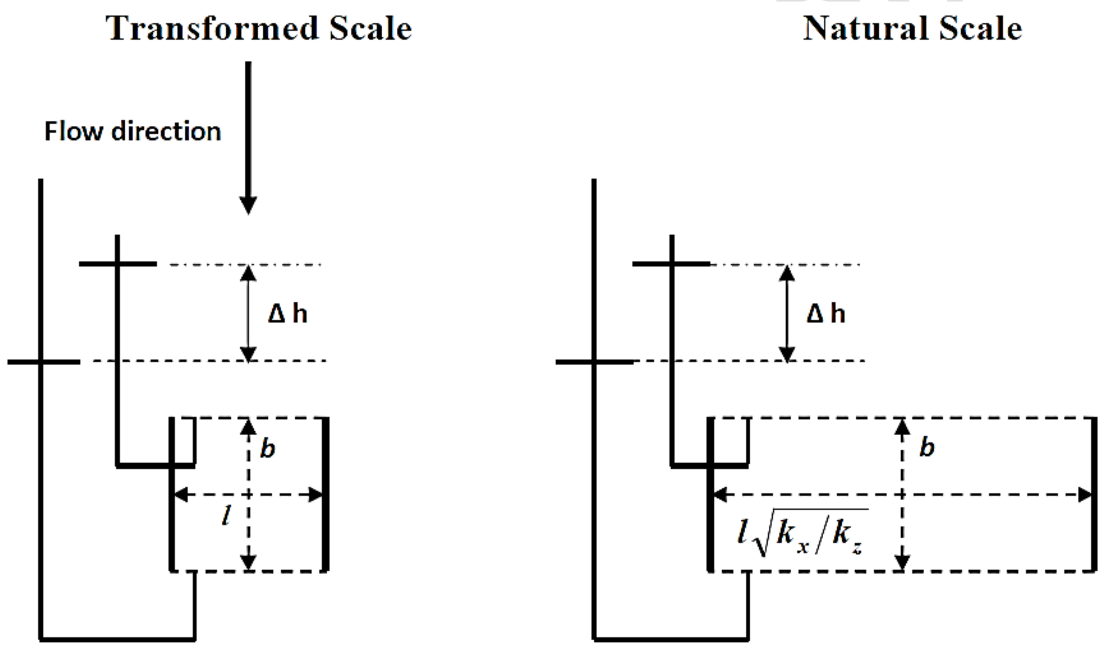
The rate of seepage per unit width can be calculated by the following equation

$$q = k_e.H.\oint = \sqrt{k_x.k_z}.H.\frac{N_f}{N_d}$$

Where k_e effective permeability to transform the anisotropic soil to isotropic soil



$$k_e.\frac{\Delta h}{l}b(1) = k_x \frac{\Delta h}{l\sqrt{k_x/k_z}} \Rightarrow k_e = \sqrt{k_x.k_z}$$



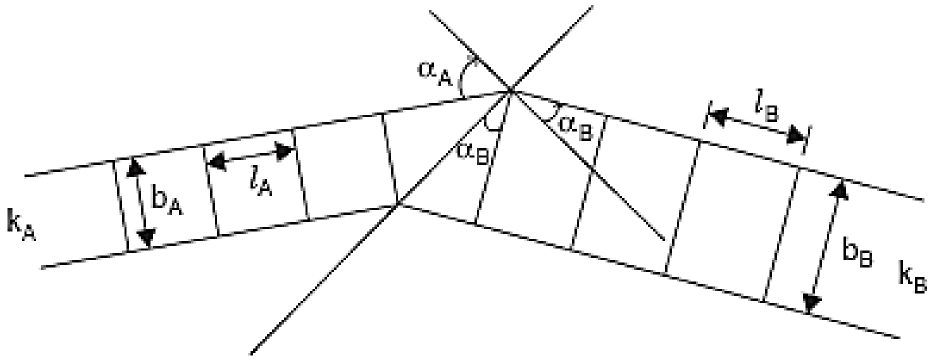
$$k_e \frac{\Delta h}{b} l(1) = k_z \frac{\Delta h}{b}.l\sqrt{k_x/k_z} \Rightarrow k_e = \sqrt{k_x.k_z}$$

5.16 Transfer Condition

In case of flow perpendicular to soil strata, the loss of head and rate of flow are influenced primarily by the less pervious soil whereas, in the case of flow parallel to the strata, the rate of flow is essential controlled by comparatively more pervious soil.

The following shows a flow channel (part of two – dimensional flow net) going from soil A to soil B with kA ¹ kB (two layers). Based on the principle of continuity, i.e., the same rate of flow exists in the flow channel in soil A as in soil B, we can derive the relationship between the angles of incident of the flow paths with the boundary for the two flow channels. Not only does the direction of flow change at a boundary between soils with different permeabilities, but also the geometry of the figures in the flow net changes.

As can be seen in the figure below, the figures in soil B are not squares as is the case in soil A, but rather rectangles.



$$\Delta q_A = \Delta q_B$$

$$\Delta q_A = k_A \frac{\Delta h}{l_A} b_A$$

$$\Delta q_B = k_B \frac{\Delta h}{l_B} b_B$$

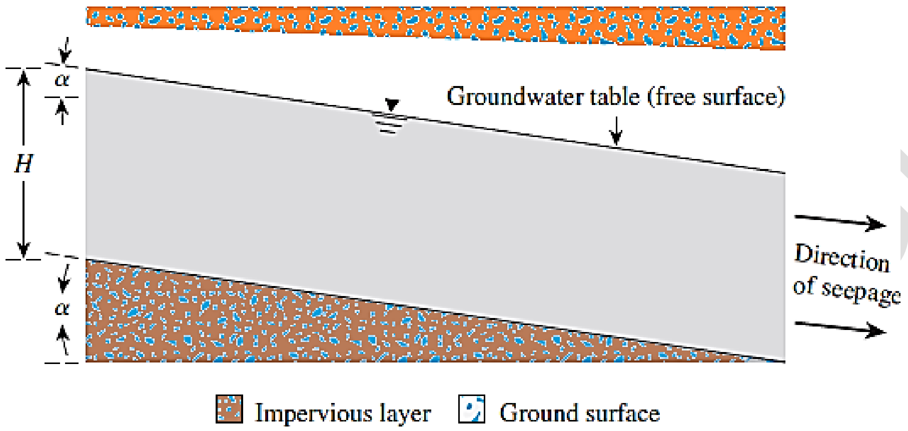
$$k_A \frac{\Delta h}{l_A} b_A = k_B \frac{\Delta h}{l_B} b_B$$

$$\frac{l_A}{b_A} = \tan\alpha_A \cdots and \cdots \frac{l_B}{b_B} = \tan\alpha_B$$

$$\frac{k_A}{\tan\alpha_A} = \frac{k_B}{\tan\alpha_B} \Rightarrow \frac{k_A}{k_B} = \frac{\tan\alpha_A}{\tan\alpha_B}$$

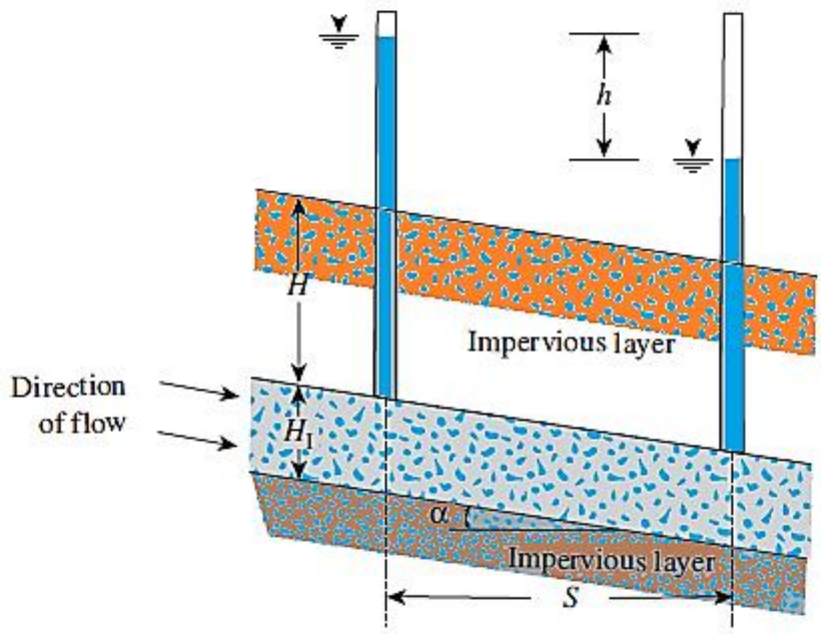
Homework Chapter (5)

5.1 A permeable soil layer is underlain by an impervious layer as shown in Figure. Knowing that $k = 4.8*10^{-3}$ cm/sec for the permeable layer, calculate the rate of seepage through this layer in m^3/hr/m width. Given: H = 4.2 m and $\alpha = 6°$. **Ans. $Q \approx 7.54\times10^{-2}$ m^3/hr/m**



5.2 Find the rate of flow in m^3/sec/m (at right angles to the cross section shown in Figure, through the permeable soil layer. Given: H = 4 m, H_1 = 2 m, h = 2.75 m, S = 30 m, α = 14°, and k = 0.075 cm/sec.

Ans. $Q = 1.29\times10^{-4}$ m^3 /sec/m



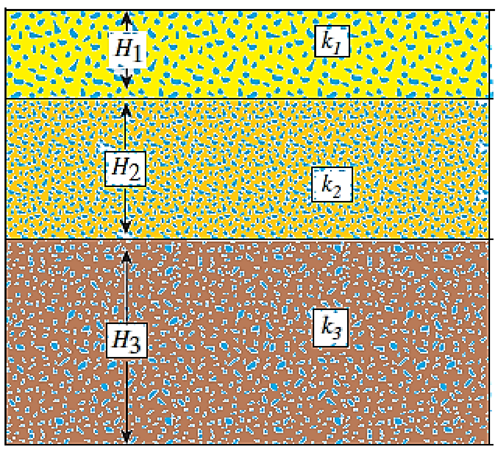
5.3 A layered soil is shown in Figure. Given that

H_1 = 1 m $k_1 = 10^{-4}$ cm/sec

H_2 = 1 m $k_2 = 2.8*10^{-2}$ cm/sec

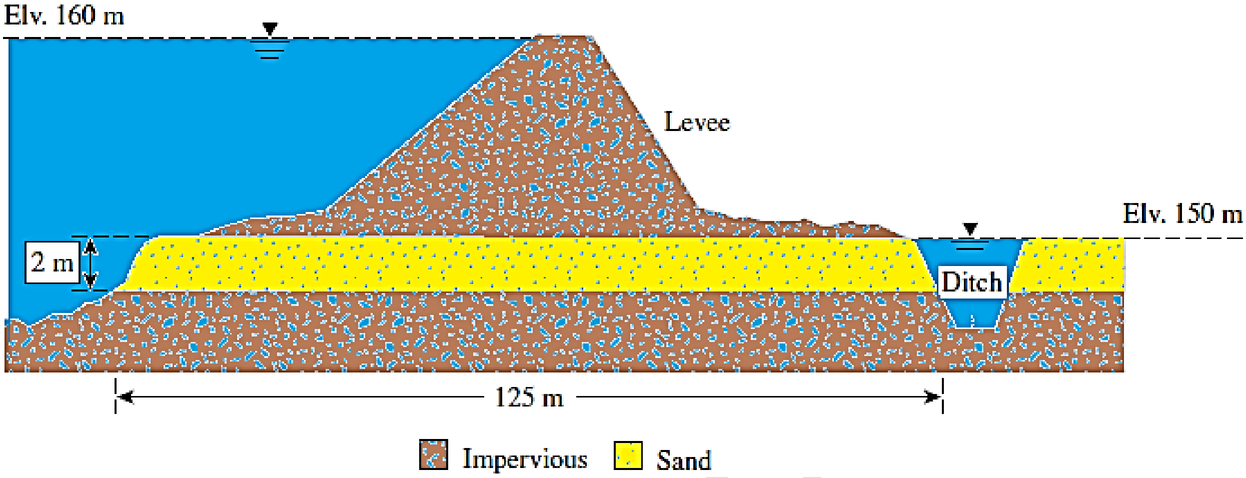
H_3 = 2 m $k_3 = 3.510^{-5}$ cm/sec

Estimate the ratio of equivalent permeability, $k_{H(eq)}/k_{V(eq)}$. **Ans. 118.32**



5.4 The figure shows the cross-section of a levee that is 500 m long and is underlain by a 2-m-thick permeable sand layer. It was observed that the quantity of water flowing through the sand layer into the collection ditch is 250 m^3/day. What is the hydraulic conductivity of the sand layer.

Ans. k = 3.125 m/day



5.5 Consider the setup shown in Figure in which three different soil layers are located inside a cylindrical tube of diameter 150 mm. A constant-head difference of 470 mm is maintained across the soil sample. The porosities and hydraulic conductivities of the three soils in the direction of the flow are as given in the table:

Soil	n	k (cm/sec)
I	0.5	$5*10^{-3}$
II	0.6	4.210^{-2}
III	0.33	3.910^{-4}

a. Determine the quantity of water flowing through the sample per hour.

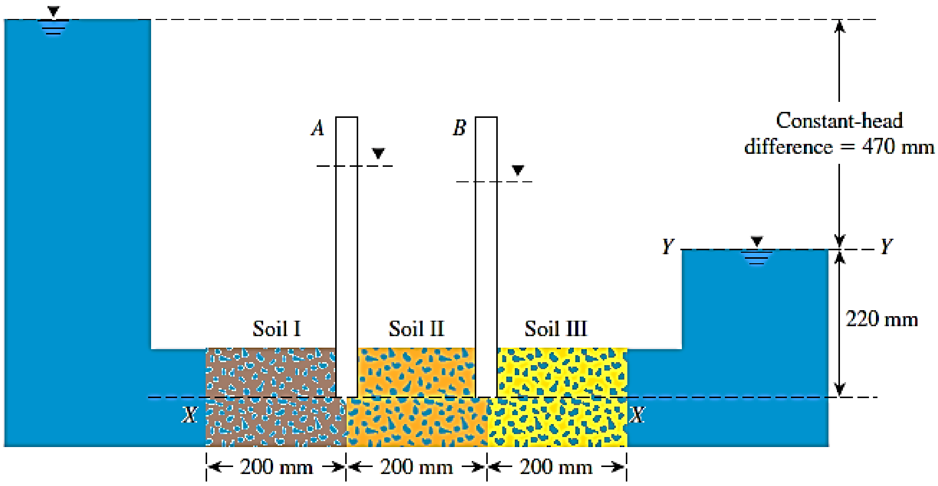
b. Denoting the downstream water level (Y-Y) to be the datum, determine the elevation head (h_e), pressure head (h_p), and the total head (h_t) at the entrance and exit of each soil layer.

c. Plot the variation of the elevation head, pressure head and the total head with the horizontal distance along the sample axis (X-X).

d. Plot the variations of discharge velocity and the seepage velocity along the sample axis.

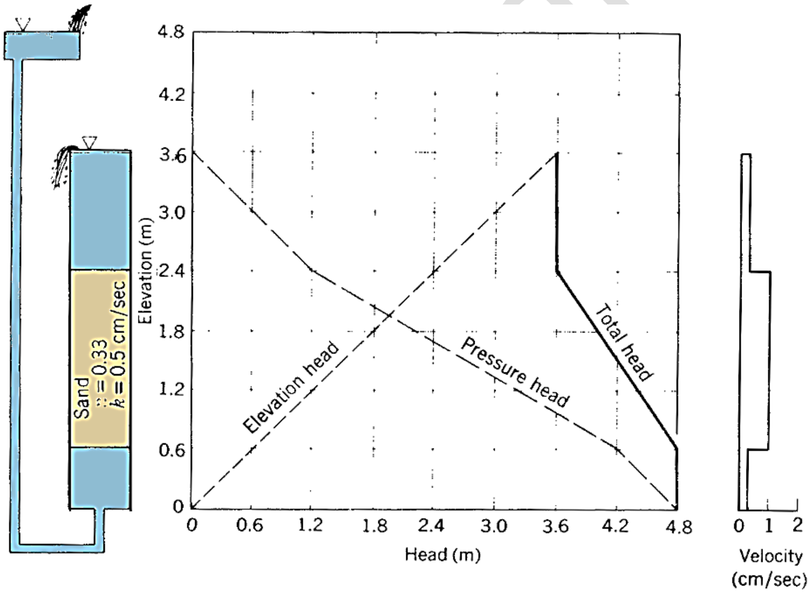
e. What will be the height of the vertical columns of water inside piezometers A and B installed on the sample axis.

Ans.: (a) 536.4 cm^3/hr., (b) 220mm (d) at A = 656.29 mm at B = 656.29 mm



5.6 For the flow situation shown in Figure, compute the vertical effective stress in the sand at elevation +1.2 m Gs = 2.60 and S = 100%.

Ans.: σ′$_v$ = 0.844 kN/m^2



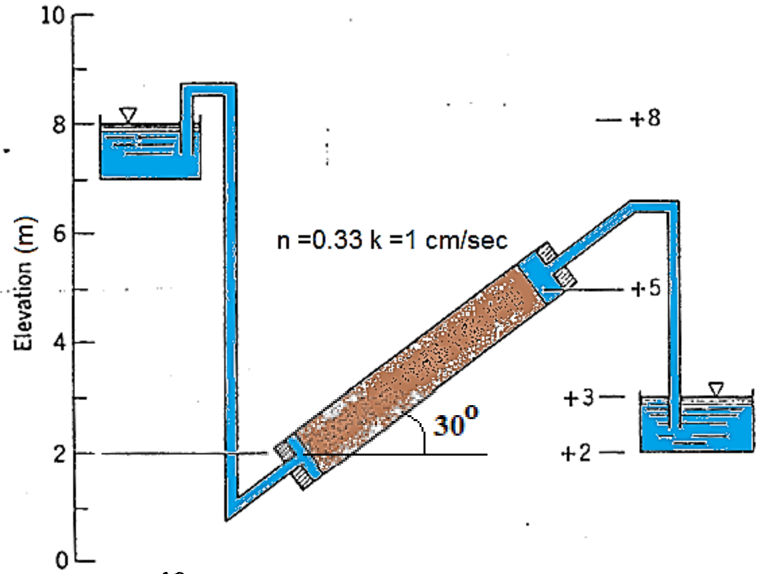
5.7 In a certain sand deposit, with γ_{sat} = 20 kN/m^3, the water table is at the ground surface. Compute the total stress, pore pressure, and effective stress on a horizontal plane at a depth of 4.5 m for each of the following cases: a. Static groundwater. b. Upward flow under a gradient of 0.5,

Ans.: (a) σ′$_v$ = 45.86 kN/m^2, (b) σ′$_v$ = 23.78 kN/m^2

5.8 A jar 100 cm high and 10 cm^2 in cross-sectional area is filled with soil and water having an overall average unit weight of 10.57 kN/m^3. The specific gravity of the soil is 2.80. For each of the following three cases compute σv, u, and, σ'v, at the bottom of the jar:

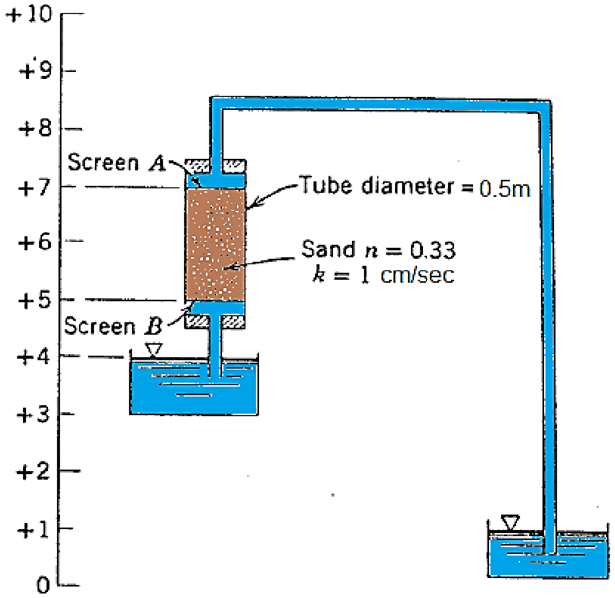
a. Uniform slurry. b. The sediment of soil 5 cm thick and seawater, γ_w = 10.07 kN/m^3. c. The sediment of soil 6 cm thick and pure water, γ_w = 9.81 kN/m^3. For cases (b) and (c) compute the void ratio of the sediments. **Ans.: (a) σ$_v$ = 10.57 kN/m^2, u = 9.81 kN/m^2, σ′$_v$ = 0.76 kN/m^2, (b) σ$_v$ = 10.567 kN/m^2, u = 10.07 kN/m^2, σ′$_v$ = 0.497 kN/m^2, e = 0.8255, (c) σ$_v$ = 10.57 kN/m^2, u = 9.81 kN/m^2, σ′$_v$ = 0.76 kN/m^2, e = 0.412**

5.9 For the setup shown. Plot to scale elevation head, pressure head, total head and seepage velocity versus distance along the sample axis.

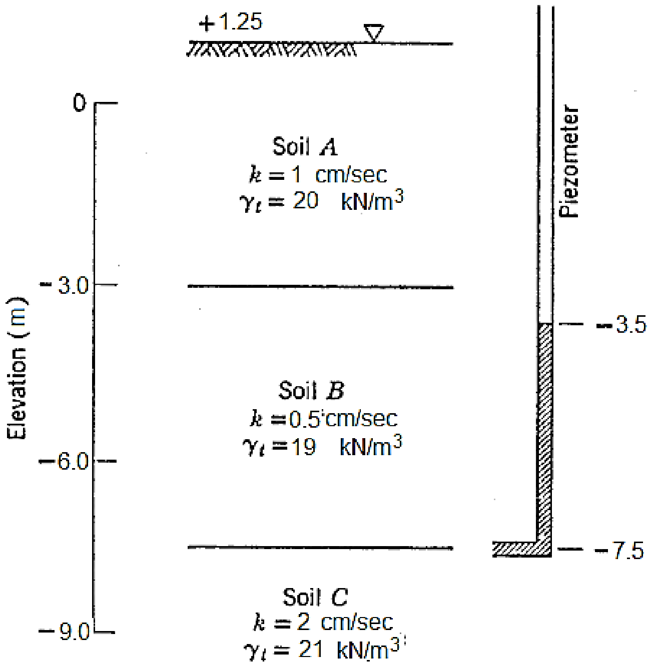


5.10 For the setup shown, compute the vertical force exerted by the soil on screen A and that on screen B. Neglect friction between the soil and tube. G_s= 2.75.

Ans.: @A = 11.55kN↑, @B = 6.44kN↓



5.11 For the set up shown: Steady vertical seepage is occurring. Make a scaled plot of elevation versus pressure head, pressure head, seepage velocity, and vertical effective stress. Determine the seepage force on a 1 m cube whose center is at elevation -4.5 m. G_s for all soils = 2.75

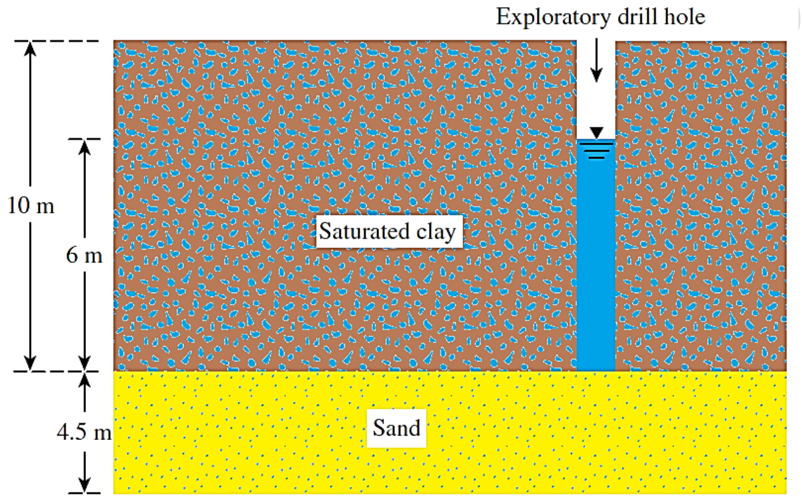


5.12 For a sandy soil with G_s = 2.68, calculate the critical hydraulic gradient that will cause boiling or quick condition for e = 0.38, 0.48, 0.6, 0.7, and 0.8. Plot the variation of i_{cr} with the void ratio.

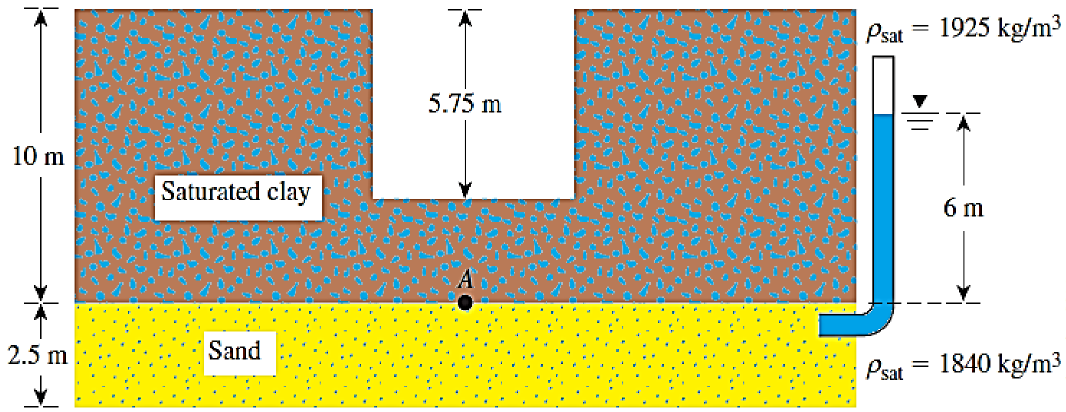
Ans.:

e	0.38	0.48	0.6	0.7	0.8
i_c	1.22	1.14	1.05	0.99	0.93

5.13 An exploratory drill hole was made in a stiff saturated clay having a moisture content of 29% and G_s = 2.68. The sand layer underlying the clay was observed to be under artesian pressure. Water in the drill hole rose to a height of 6 m above the top of the sand layer. If an open excavation is to be made in the clay, determine the safe depth of excavation before the bottom heaves. **Ans.: H =6.91m**

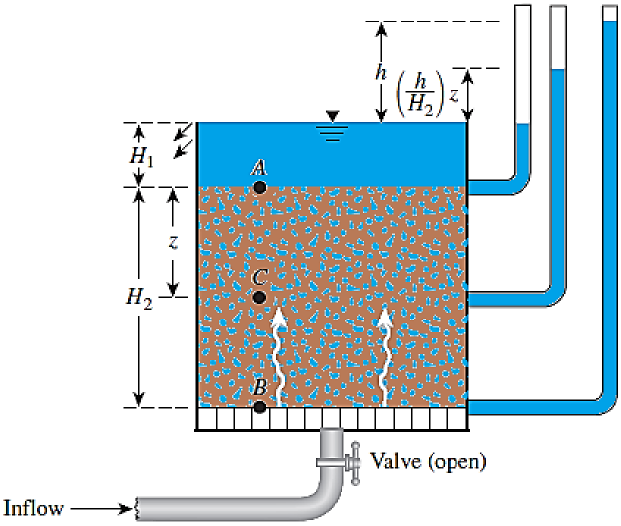


5.14 A 10-m-thick layer of stiff saturated clay is underlain by a layer of sand. The sand is under artesian pressure. A 5.75-m-deep cut is made in the clay. What would be the maximum permissible depth of cut before heaving would occur. Determine the factor of safety against heaving at point A. **Ans.: H =6.88m, FOS = 1.36**



5.15 Refer to Problem 5.14. Water may be introduced into the cut to improve the stability against heaving. Assuming that a cut is made up to the maximum permissible depth calculated in Problem 5.14, what would be the required height of water inside the cut in order to ensure a factor of safety of 1.5? **Ans.: H =3.0m**

5.16 Refer to Figure in which upward seepage is taking place through a granular soil contained in a tank. Given: H_1 = 1.5 m; H_2 = 2.5 m; h = 1.5 m; area of the tank = 0.62 m^2; void ratio of the soil, e = 0.49; G_s = 2.66; and hydraulic conductivity of the sand (k) = 0.21 cm/sec.



a. What is the rate of upward seepage?

b. Will boiling occur when h =1.5 m? Explain.

c. What would be the critical value of h to cause boiling?

Ans.: (a) q = 781.2 cm^3/sec, (b) No. (c) h = 2.77m

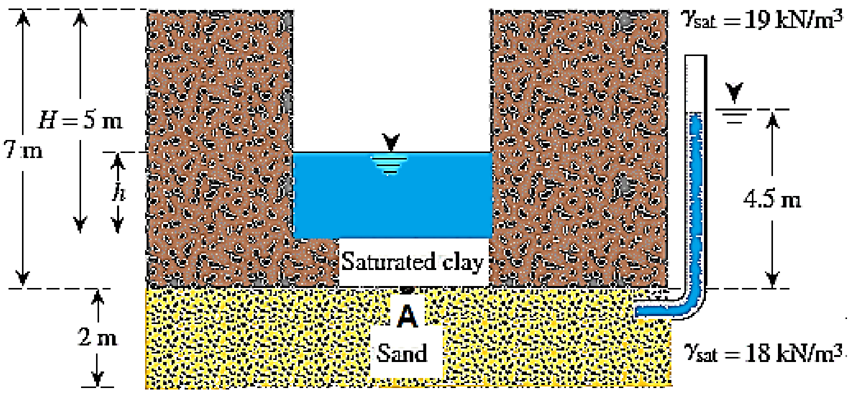
5.17 Refer to Figure of Problem 5.16. If H_1 = 0.91 m, H_2 = 1.37 m, h = 0.46 m, γ_{sat} = 18.67 kN/m^3, area of the tank = 0.58 m^2, and hydraulic conductivity of the sand (k) = 0.16 cm/sec,

a. What is the rate of upward seepage of water (m^3/min)?

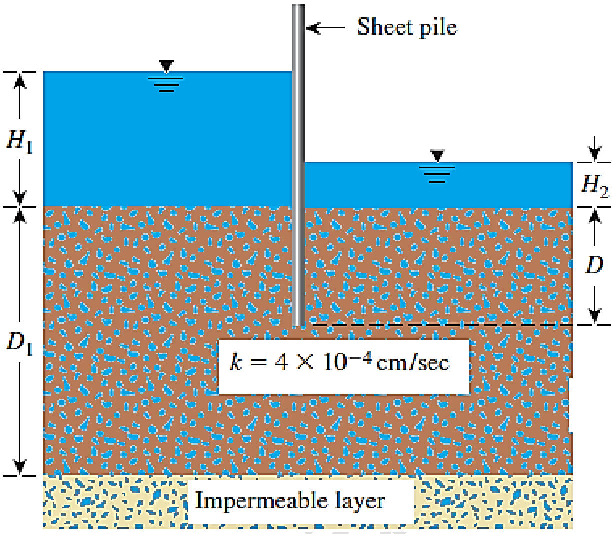
b. If the point C is located at the middle of the soil layer, then what is the effective stress at C? **Ans.: (a) i = 0.336, (b) q = 0.0187 m^3/min**

5.18 A cut is made in a stiff, saturated clay that is underlain by a layer of sand. What should be the height of the water, h, in the cut so that the stability of the saturated clay is not lost?

Ans.: (a) h = 0.63



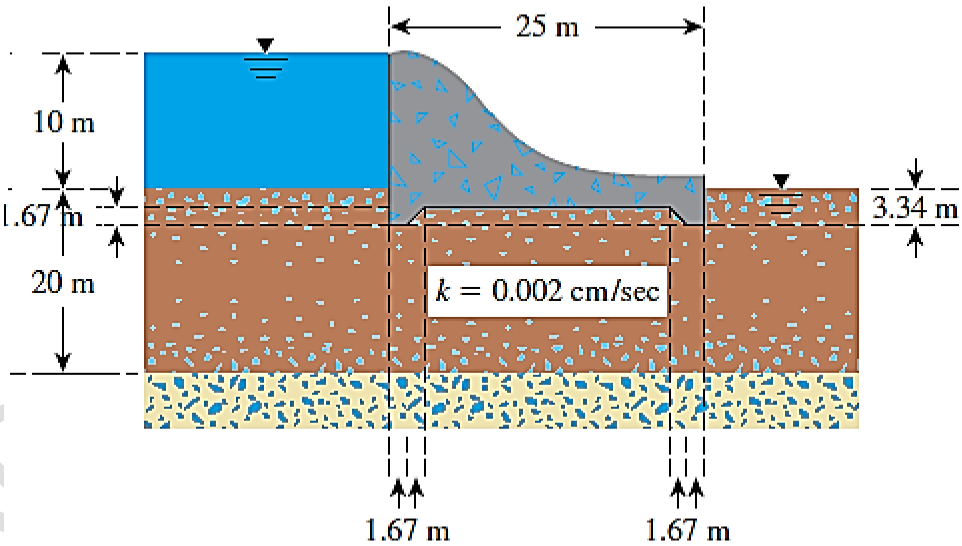
5.19 Refer to figure. Given: H_1 = 6 m, D = 3 m, H_2 = 1.5 m, D_1 = 6 m, draw a flow net. Calculate the seepage loss per meter length of the sheet pile (at a right angle to the cross section shown). **Ans.: N_f = 4, N_d = 8, q = 77.76*10^-6^ m³/day/m**



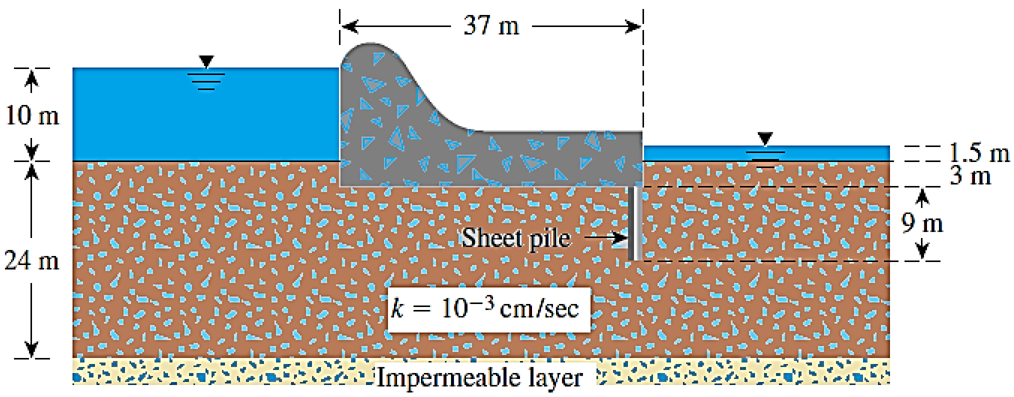
5.20 Draw a flow net for the single row of sheet piles driven into a permeable layer as shown in Figure. Given:

- H_1 = 3 m, D = 1.5 m
- H_2 = 0.5 m, D_1 = 3.75 m, **Ans.: N_f = 3, N_d = 5, q = 0.518 m³/day/m**

5.21 For the hydraulic structure shown in Figure, draw a flow net for flow through the permeable layer and calculate the seepage loss in m³/day/m. Calculate the hydraulic uplift force at the base of the hydraulic structure per meter length (measured along the axis of the structure). **Ans.: N_f = 5, N_d = 12, q = 7.2 m³/day/m**



5.22 Draw a flow net for the weir shown in Figure. Calculate the rate of seepage under the weir. **Ans.: N_f = 4, N_d = 14, q = 2.1 m³/day/m**



5.23 The figure shows a concrete dam. Consider Case 1 without the sheet pile and Case 2 with the sheet pile along the upstream side.

a. Draw flow nets for both cases.

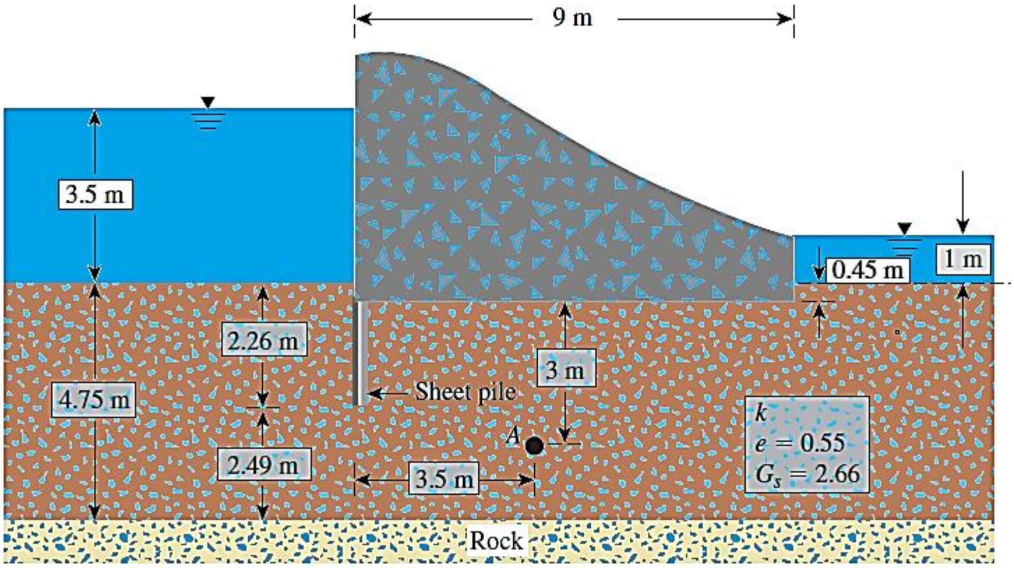
b. Determine the value of for both cases. (Note: q = m^3/s/m; k = m/s.)

c. Determine the factor of safety (FS) against heaving, for Cases 1 and 2. Comment on any differences in the magnitude of FS.

d. Estimate the seepage force (kN/m^3) at point A in the direction of seepage for Cases 1 and 2. Comment on any difference in the magnitude of the seepage force.

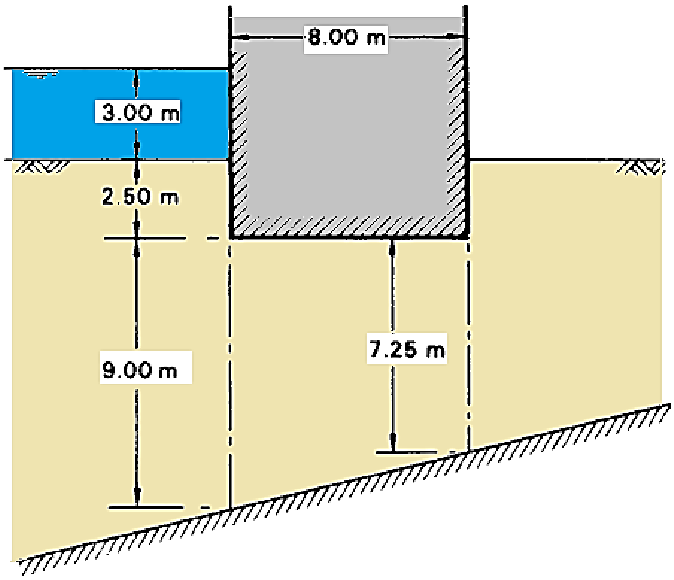
Ans.: case 1: N_f = 4, N_d = 11, q = 0.909 m^3/day/m, FS = 1.67, seepage force = 1.4 kN/m^3

case 2: N_f = 3.5, N_d = 13, q = 0.673 m^3/day/m, FS = 1.97, seepage force = 1.33 kN/m^3



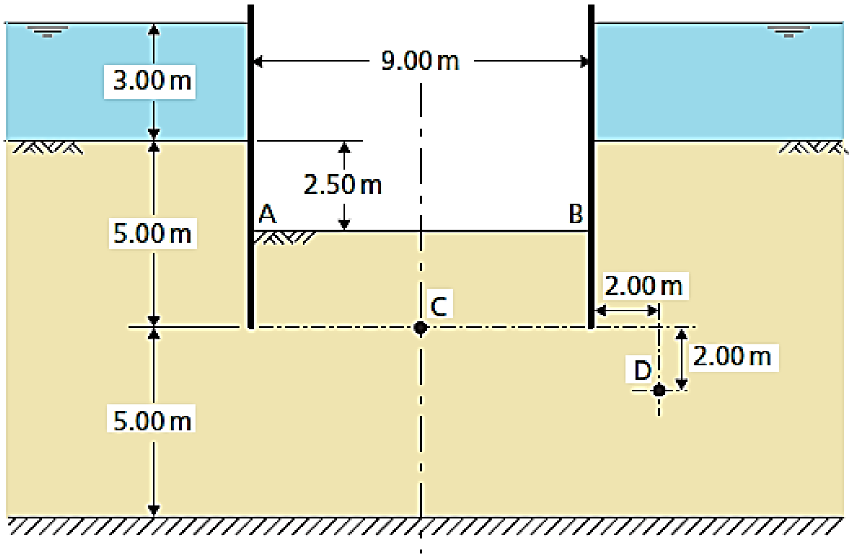
5.24 Draw the flow net for seepage under the structure detailed in Figure and determine the quantity of seepage. The coefficient of permeability of the soil is 5*10^{-5} m/s. What is the uplift force on the base of the structure.

Ans.: N_f = 3.5, N_d = 9, q = 5.8*10^{-5} m^3/sec/m



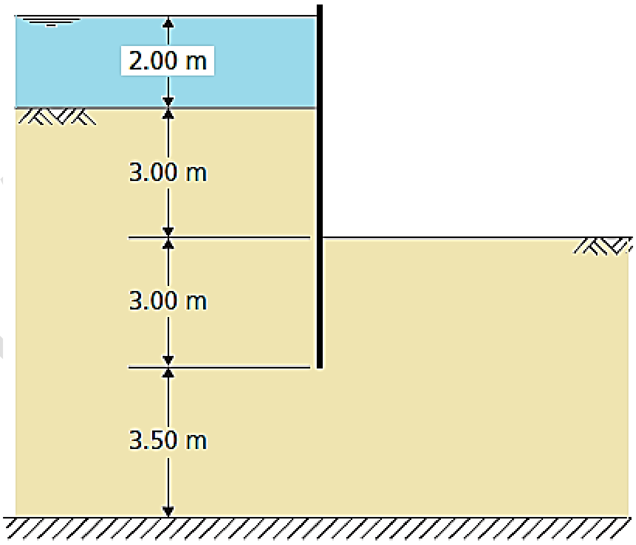
5.25 The section through a long cofferdam is shown in Figure, the coefficient of permeability of the soil being $4*10^{-7}$ m/s. Draw the flow net and determine the quantity of seepage entering the cofferdam.

Ans.: N_f = 10, N_d = 11, q = $4*10^{-6}$ m^3/sec/m



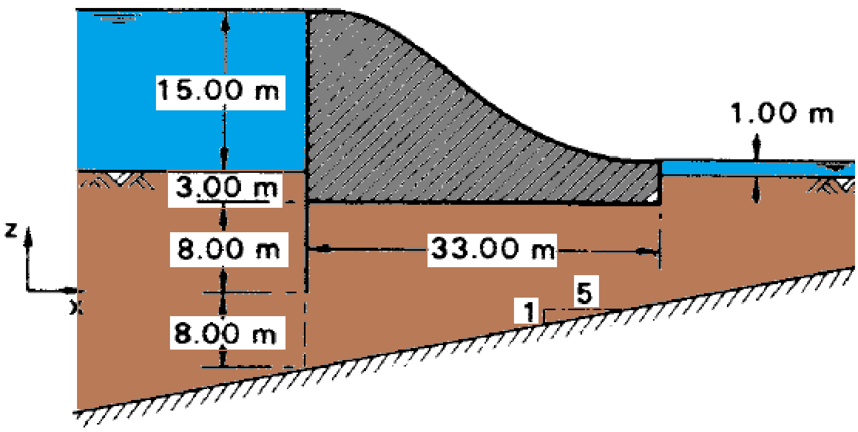
5.26 The section through sheet pile is shown in Figure, the coefficient of permeability of the soil is 2.0 × 10–6 m/s. Draw the flow net and determine the quantity of seepage.

Ans.: N_f = 4.2, N_d = 9, q = $4.7*10^{-6}$ m^3/sec/m

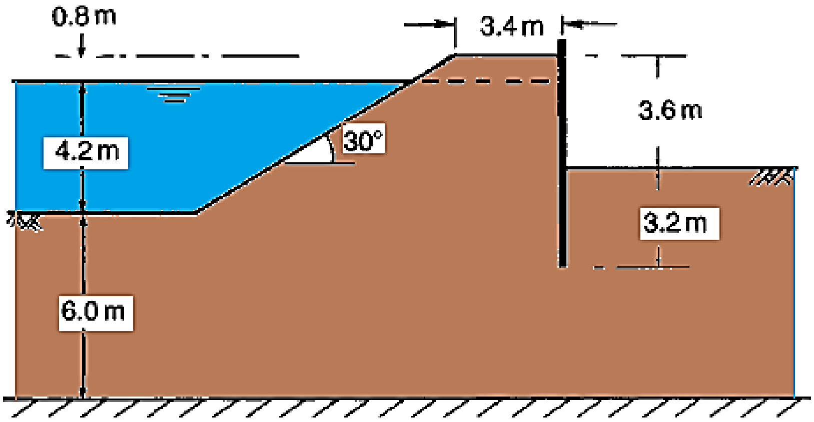


5.27 The dam shown in Figure is located on the anisotropic soil. The coefficients of permeability in the x and z directions are $5*10^{-7}$ and $1.8*10^{-7}$ m/s, respectively. Determine the quantity of seepage under the dam.

Ans.: N_f = 3.25, N_d = 12, q = $1.1*10^{-6}$ m^3/sec/m



5.28 Details of an excavation adjacent to a canal are shown in Figure Determine the quantity of seepage into the excavation if the coefficient of permeability is 4.5*10-5 m/s. **Ans.: N_f = 3.3, N_d = 7, q = 5.9*10^{-5} m^3/sec/m**



5.29 Determine the quantity of seepage under the dam shown in section in Figure. Both layers of soil are isotropic, the coefficients of permeability of the upper and lower layers being 2.0*10^{-6} and 1.6*10^{-5} m/s, respectively. **Ans.: N_f = 5.6, N_d = 11, q = 1.0*10^{-5} m^3/sec/m**

5.30 Using the flow net shown below in Figure. Compute the uplift pressures acting on the bottom of the concrete dam and develop a plot.